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THE EFFECT OF DISPERSION IN
TRAJECTORY PARAMETERS ON
A NOMINAL MARTLET IV TYPE ORBIT

by

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TABLE OF CONTENTS

1.	INTRODUCTION	
2.	NOMINAL TRAJECTORY	
2.1	Nominal Mission and Trajectory Parameters	2.1
2.2	Estimated Dispersion in the Trajectory Parameters	2.5
3.	ORBIT SENSITIVITY TO DISPERSION IN TRAJECTORY PARAMETERS	
3.1	Phase I	3.1
3.1.1	Gun Muzzle Velocity (P 1.1)	3.1
3.1.2	Gun Elevation (P 1.2)	3.4
3.2	Phase 2 - First Stage	3.7
3.2.1	Ignition Time (P 2.1)	3.7
3.2.2	Fuel Specific Impulse (P 2.4)	3.10
3.2.3	Burning Rate (P 2.5)	3.10
3.2.4	Weight of Fuel (P 2.6)	3.13
3.3	Phase 3 - Second Stage	3.18
3.3.1	Ignition Time (P 3.1)	3.18
3.3.2	Firing Elevation (P 3.2)	3.19
3.3.3	Firing Azimuth (P 3.3)	3.22
3.3.4	Specific Impulse (P 3.4)	3.27
3.3.5	Burning Rate (P 3.5)	3.27
3.3.6	Weight of Fuel (P 3.6)	3.32
3.4	Phase 4 - Third Stage	3.35
3.4.1	Ignition Time (P 4.1)	3.35
3.4.2	Firing Elevation (P 4.2)	3.35
3.4.3	Firing Azimuth (P 4.3)	3.36
3.4.4	Specific Impulse (P 4.4)	3.42
3.4.5	Burning Rate (P 4.5)	3.42
3.4.6	Weight of Fuel (P 4.6)	3.42
4.	DISCUSSION AND OPTIMIZATION	
4.1	Discussion of the Results	4.1
4.2	Orbit Optimization	4.12
5.	CONCLUSIONS	
	Appendix A: Nominal Trajectory	

Appendix B: Orbit Parameters Sensitivity to Payload
Weight Variations

Appendix C: Graphical Summary of the sensitivity of the
orbit parameters to the trajectory parameters

1.0 INTRODUCTION

The purpose of this study is to assess the effects of variation in the parameters which govern the orbit-insertion trajectories of satellites launched using a Martlet IV type vehicle. The object in doing this is to reveal those parameters which have the greatest influence and over which control must be arranged in order to ensure injection into the orbit required by a particular mission. The results of this analysis should also provide an indication of the likelihood of attaining a specified orbit.

The launch vehicle and its capabilities are discussed in references 1, 2 and of the various families of trajectories possible, the one selected as the basis for this study is the low-gun-angle launch of the all-solid rocket configuration. Figure 6 Reference 1 shows a nominal trajectory in this category identified as "Case 1046". This study is confined to that trajectory and caution is advisable in extrapolating the results to include launches having significantly different initial conditions or to gun-launched vehicles of other configurations.

The trajectory under investigation has been divided into four phases: gun launch and first glide, first stage ignition, second stage ignition, second glide and third stage ignition. Each phase is defined by a number of parameters which when set at a specific value define an orbit or a trajectory. The outstanding characteristic of a Martlet IV trajectory is of course that the launch phase is

performed by firing the unignited rocket from a 16" gun imparting to it a large initial velocity along a very precise trajectory prior to first stage ignition at a predetermined time.

The nominal trajectory mentioned above was computed using the methods developed by McKee in Reference (3) and had to be modified slightly to suit this study. This is discussed in greater detail in the next section; here it will suffice to say that the modified nominal trajectory chosen yielded an almost circular orbit at 400 nautical miles (Figure 1, Appendix A). Each of the parameters in the four phases of the trajectory was then varied about its nominal value and a new trajectory computed. Since the characteristics of greatest importance were the perigee and apogee, these have been plotted to show the sensitivity to dispersion in each parameter. The curves are contained in Part 3 and Appendices B and C of this report.

Although the analysis deals with a hypothetical mission whose nominal orbit altitude is approximately 400 nautical miles, it will be seen that the summary graphs of Appendix C have been plotted to include perigees of 200 miles. Simple extrapolation enables one to assess the relatively wide variation in parameters which could be tolerated if the mission requirement were to be relaxed to accept an eccentric orbit with a perigee of 100 nautical miles.

2.0 NOMINAL TRAJECTORY

2.1 Nominal Mission and Trajectory Parameters

As mentioned in the introduction, the optimized trajectory 1046 (Orbital and High Altitude Probing Potential of Gun Launched Rockets by G. V. Bull, D. Lyster, G. V. Parkinson, (R-SKI-W-R-13) page 15 Fig. 6) given by R. M. McKee's computer program had to be modified in order to be able to vary meaningful trajectory parameters.

The hypothetical mission described by trajectory 1046 uses a Martlet IV rocket in all-solid configuration to place a 75 lb payload in a circular orbit at approximately 700 km (375 naut. miles). It is assumed that the launch vehicle is spin-stabilized at launch and the attitude control system serves only to re-orient the third stage before its ignition.

A careful investigation of the pertinent parameters has shown that the following parameters were controllable:

- 1st stage ignition time
- 2nd stage ignition time
- 3rd stage ignition time
- 3rd stage pitch angle
- 3rd stage yaw angle

Unfortunately many other parameters have been deemed significant although not controllable during the flight:

Gun Muzzle Velocity

Gun Elevation

1st stage Fuel Specific Impulse

1st stage Fuel Burning Rate (constant fuel weight)

1st stage Fuel Weight (constant burning rate and structural weight)

2nd stage Fuel Specific Impulse

2nd stage Fuel Burning Rate (constant fuel weight)

2nd stage Fuel Weight (constant burning rate and structural weight)

2nd stage Firing Pitch Angle

2nd stage Firing Yaw Angle

3rd stage Fuel Specific Impulse

3rd stage Fuel Burning Rate (constant fuel weight)

The above listed parameters are all either measurable or adjustable before or during the flight for compliance with specifications except for:

2nd stage Firing Pitch Angle

2nd stage Firing Yaw Angle.

These parameters are of particular concern because of the possibility of tip-off error occurring at stage separation. Trajectory case 1046 allowed no time interval and for the modified trajectory used in this analysis a minimum practical interval of 0.5 second was stipulated as the time required between first stage burn-out and second stage ignition.

In order to facilitate the study and discussion of the sensitivity of orbit parameters to the above trajectory parameters, it was found convenient to subdivide the flight into four phases:

TABLE 2.1

phase 1:	<u>Gun Parameters</u>
	P 1.1 Gun Muzzle Velocity
	P 1.2 Gun Elevation
phase 2:	<u>1st stage Parameters</u>
	P 2.1 Ignition Time
	P 2.4 Fuel Specific Impulse
	P 2.5 Fuel Burning Rate
	P 2.6 Fuel Weight
phase 3:	<u>2nd stage Parameters</u>
	P 3.1 Ignition Time
	P 3.2 Firing Pitch Angle
	P 3.3 Firing Yaw Angle
	P 3.4 Fuel Specific Impulse
	P 3.5 Fuel Burning Rate
	P 3.6 Fuel Weight
phase 4:	<u>3rd stage Parameters</u>
	P 4.1 Ignition Time
	P 4.2 Firing Pitch Angle
	P 4.3 Firing Yaw Angle
	P 4.4 Fuel Specific Impulse
	P 4.5 Fuel Burning Rate
	P 4.6 Fuel Weight

The Nominal Trajectory was consequently constructed from the trajectory 1046 with the appropriate changes in the ignition and firing indicators and parameters as indicated in Table 2.2. Nevertheless it was not possible to match trajectory 1046 with the parameters defined in Table 2.1; for instance the second stage firing attitude which was assumed parallel to the velocity vector in trajectory 1046 was now fixed (elevation 27.08 deg., azimuth 3.33 deg.) during the burning of the fuel. Such unmatched differences resulted in a slightly

different orbit and required a new optimization; this was done with respect to the gun elevation and the third stage ignition. The following Table gives an idea of the modifications effected to the original trajectory 1046:

TABLE 2.2		
Event	Trajectory 1046	New Nominal Traj.
1st stage ignition	40 km	14.61 sec from launch
2nd stage ignition	0.5 sec after 1st stage B.O.	30.11 sec from launch
2nd stage firing elevation	parallel to the velocity vector	27.08 deg.
2nd stage firing azimuth	parallel to the velocity vector	3.33 deg.
3rd stage ignition	at a path angle of 0.3 deg.	604.51 sec from launch
3rd stage firing elevation	0.0 deg.	0.0 deg.
3rd stage firing azimuth	0.0 deg.	0.0 deg.

It is easy to see from Table 2.2 where the fundamental differences in the two trajectories occur; as mentioned earlier, the trajectory was re-optimized with respect to the Gun Elevation and the third stage ignition time in order to get as circular an orbit as possible, but no further attempt was made to get it perfectly circular. The new trajectory is now defined in terms of the parameters which are of interest in this analysis, that is for a "real" situation.

2.2 Estimated Dispersion in the Trajectory Parameters

What is believed to be a realistic estimate of the probable dispersion in each parameter is listed below. In some cases where insufficient data was available no figure appears.

The trajectory parameters have been grouped in three categories according to the type of error they generate:

1) Error due to the limitation to which a given parameter can be adjusted or manufactured (but not controllable from the ground during flight).

P 1.1	Gun Muzzle Velocity	: ± 200 ft/sec
P 1.2	Gun Elevation	: ± 0.5 degree
P 2.4	Fuel Specific Impulse	: ± 6 sec
P 2.5	Fuel Burning Rate	:
P 2.6	Fuel Weight	: ± 1 lbs
P 3.4	Fuel Specific Impulse	: ± 6 sec
P 3.5	Fuel Burning Rate	:
P 3.6	Fuel Weight	: ± 1 lbs
P 4.4	Fuel Specific Impulse	: ± 6 sec
P 4.5	Fuel Burning Rate	:
P 4.6	Fuel Weight	: ± 1 lbs

2) Errors due to parameters limited in accuracy but controllable during flight.

P 2.1	Ignition Time	: ± 0.5 sec
P 3.1	Ignition Time	: ± 0.5 sec
P 4.1	Ignition Time	: ± 0.5 sec
P 4.2	Firing Pitch Angle	: ± 1 deg
P 4.3	Firing Yaw Angle	: ± 1 deg

3) Unpredictable errors and uncontrollable errors occurring during flight.

P 3.2 Firing Pitch Angle	:
P 3.3 Firing Yaw Angle	:

Parameters entering in the third class deserve some comments:

P 3.2 and P 3.3 describe the vehicle attitude at second stage ignition; it is expected that an attitude error caused by tip-off occurs. It has been assumed that the interval between first stage burnout and second stage ignition would need to be kept small in order to obtain the best velocity increment from the two stages. It was thought that the interval following first stage jettison would be too short to enable a light-weight attitude control system to re-orient the spinning launch vehicle before second stage ignition and that the better procedure would be to concentrate on an interstage design which would minimize tip-off disturbances at staging. For the purpose of this study it is assumed that these disturbances would not introduce a change in the mean attitude of the spin axis of the vehicle greater than 5. degrees.

Each of the listed parameters will be varied over an appropriate range. Since our interest lies in the Perigee and the orbit eccentricity, it was found easier to retain the Perigee and the Apogee as the Orbit describing parameters.

3. ORBIT SENSITIVITY TO DISPERSION IN TRAJECTORY PARAMETERS

3.1 PHASE I

3.1.1 Gun Muzzle Velocity (P 1,1)

The results appear in Table 3.1.1 and Figure 3.1.1.

The orbit parameters seem to be fairly well optimized with respect to that parameter. A ± 200 feet per second variation in the nominal 6000 feet per second muzzle velocity results in an orbit with a minimum 262 nautical miles perigee. The variation of the perigee and apogee as a function of the gun muzzle velocity show an hyperbolic behaviour with two asymptotes; as the gun muzzle velocity is increased beyond its nominal value, the perigee remains constant slightly above 400 nautical miles while the apogee linearly increases with the muzzle velocity: for a given gun muzzle velocity V greater than 6050 feet per second, the apogee and perigee can be approximated as follows:

$$\text{Perigee} = 402 \text{ N.M.}$$

$$\text{Apogee} = 440 + \frac{250}{385} (V - 6050) \text{ N.M.}$$

Similarly for a muzzle velocity less than 5950 feet per second, the orbit parameters are given by:

$$\text{Perigee} = 360 + \frac{240}{372} (V_{\text{ft/sec}} - 5950) \text{ N.M.}$$

$$\text{Apogee} = 407 \text{ N.M.}$$

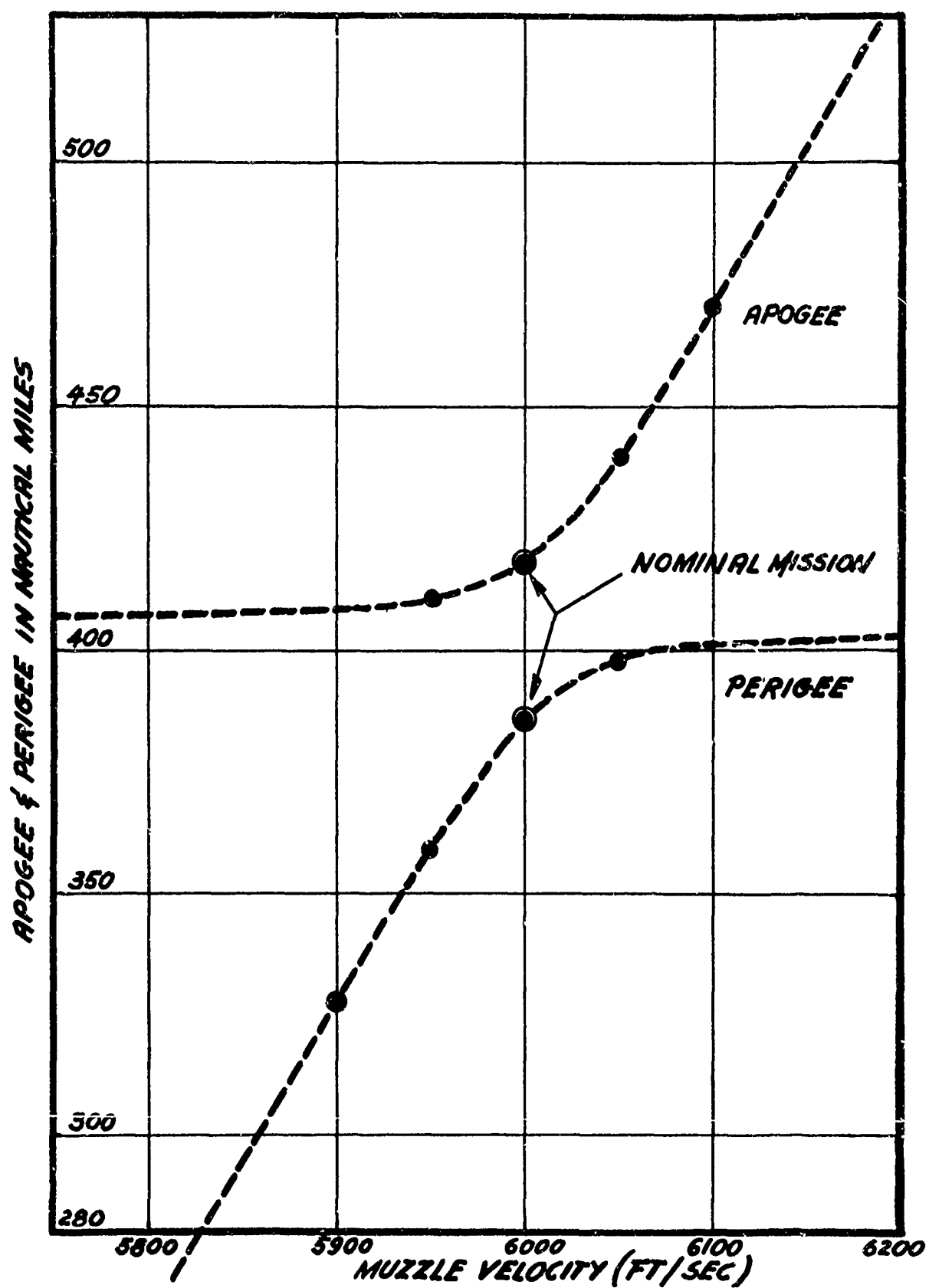


FIGURE 3.1.1

TABLE 3.1.1.1
Gun Muzzle Velocity (P 1.1) Cases 3044 - 3049

Muzzle Velocity	Vehicle Attitude 1st stg. BO		At 2nd stage ignition			Orbit Parameters	
	Elevation	Azimuth	Height	Abs. Vel.	Path Angle	Perigee	Apogee
ft/sec	degree	degree	N.M.	ft/sec	degree	N.M.	N.M.
5800	26.85	3.37	15.95	13 323	24.21	262.32	406.99
5900	26.99	3.35	16.23	13 426	24.35	327.34	408.42
5950	27.06	3.34	16.37	13 477	24.43	358.98	410.56
6000	27.13	3.33	16.51	13 529	24.50	385.52	417.77
6050	27.19	3.31	16.65	13 580	24.57	397.89	439.33
6100	27.26	3.30	16.79	13 632	24.64	401.27	470.05
6200	27.39	3.28	17.07	13 734	24.77	403.09	537.17

3.1.2 Gun Elevation (P 1.2)

The results appear in Table 3.1.2 and Figure 3.1.2. The nominal value of this parameter does not completely optimize the orbit; nevertheless, a high-sensitivity of the perigee and apogee to the variations in the gun elevation is noticed (see below).

Gun Elevation in degrees	Rate of Change in N.M./deg.	
	Apogee	Perigee
33.5	+22.67	-43.9
34.0	+35.7	-34.6

If the gun elevation is varied by $\pm .5$ degree about its nominal 34 degree value, the respective variation in apogee and perigee are as follows:

$$\text{apogee} = \pm \frac{17}{9} \text{ N.M. (Nominal : 418 N.M.)}$$

$$\text{perigee} = \pm \frac{6}{15} \text{ N.M. (Nominal : 385 N.M.)}$$

Consequently to a change of half a degree in the gun elevation corresponds a 4 percent (approx.) maximum variation in the apogee and perigee of the nominal orbit. It then goes without saying that in the $\pm .5$ degree range about its nominal value, this parameter will not affect the success of the mission.

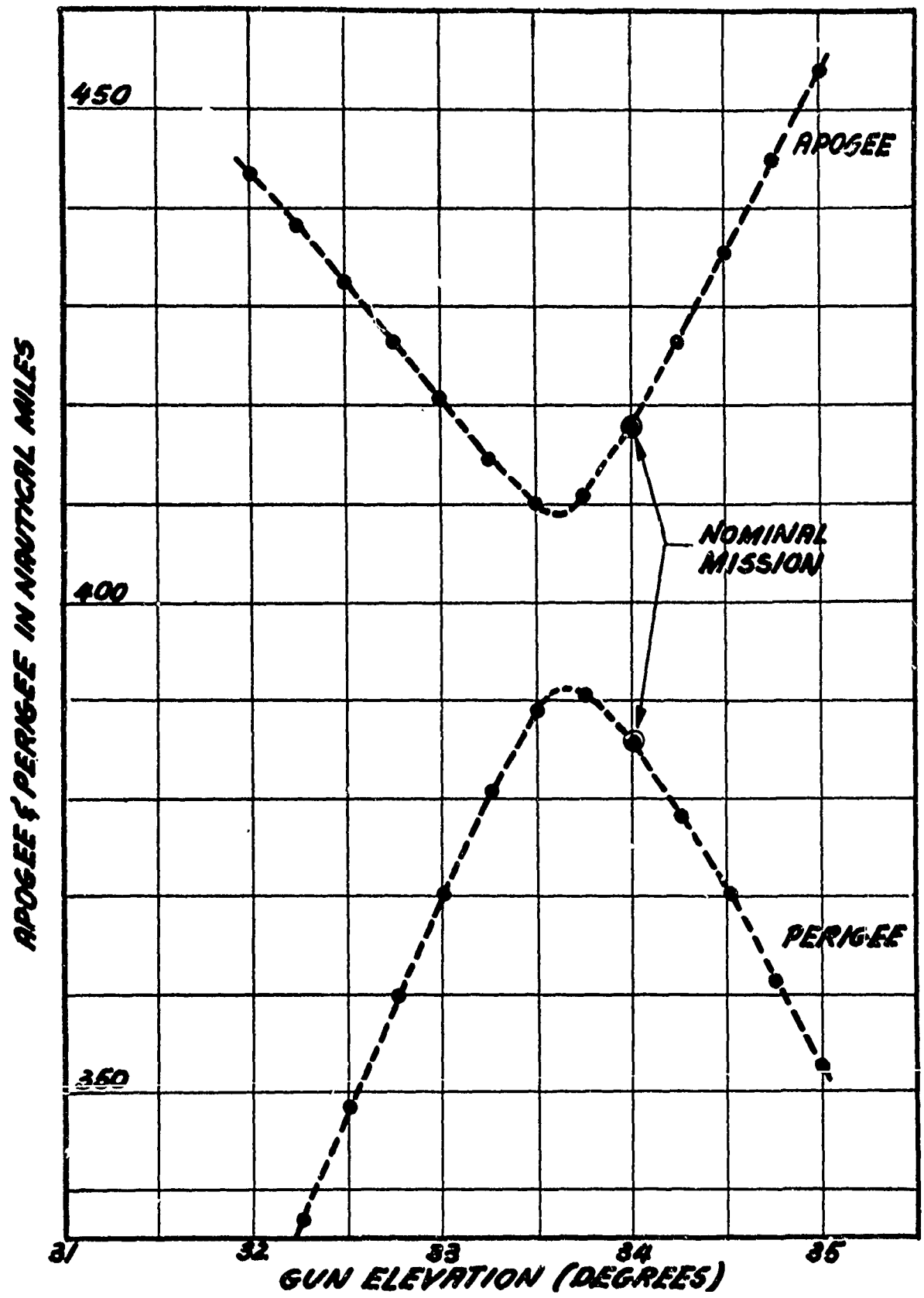


FIGURE 3.1.2

TABLE 3.1.2

Cases 3032-3042 and 3050-3054

Gun Elevation (P 1.2)

Gun Elevation	Vehicle Attitude 1st stg. BO		At 2nd stage ignition			Orbit Parameters	
	Elevation	Azimuth	Height	Abs. Vel.	Path Angle	Perigee	Apogee
degree	degree	degree	N.M.	ft/sec	degree	N.M.	N.M.
32.00	24.96	3.29	15.38	12 248	22.51	324.84	443.47
32.25	25.23	3.29	15.52	13 483	22.76	336.93	438.19
32.50	25.50	3.30	15.67	13 490	23.01	348.01	432.57
32.75	25.77	3.30	15.81	13 497	23.25	359.83	426.67
33.00	26.04	3.30	15.95	13 504	23.50	370.56	420.58
33.25	26.31	3.31	16.09	13 511	23.75	380.47	414.66
33.50	26.59	3.31	16.23	13 517	24.00	388.36	410.10
33.75	26.86	3.32	16.37	13 523	24.25	390.25	410.91
34.00	27.13	3.33	16.51	13 529	24.50	385.52	417.77
34.25	27.40	3.33	16.65	13 534	24.75	378.33	426.53
34.50	27.67	3.34	16.79	13 539	25.00	370.18	435.66
34.75	27.94	3.34	16.93	13 544	25.24	361.52	444.83
35.00	28.22	3.35	17.07	13 549	25.49	352.44	453.93

3.2 PHASE 2

3.2.1 First Stage Ignition Time (P 2.1)

The results appear in Table 3.2.1 and Figure 3.2.1. Nominally fixed at 14.61 second in order to get the ignition at 40 kilometers above the sea level, this parameter does not optimize the orbit parameters. An ignition time of 16 seconds, for instance, would have improved the eccentricity of the orbit, brought the perigee from 385 up to 408 nautical miles and, more important, reduced the sensitivity of the apogee and the perigee to this parameter.

To a variation of half a second in the nominal 14.61 second ignition time correspond the following maximum changes in the apogee and perigee:

$$\text{perigee} = \begin{matrix} + 9 \\ - 7 \end{matrix} \text{ N.M. (nominal 385.5 N.M.)}$$

$$\text{apogee} = \begin{matrix} + 1.75 \\ - 1.25 \end{matrix} \text{ N.M. (nominal 417.8 N.M.) ;}$$

approximately 2 percent change in the perigee and only half a percent change in the apogee results from the $\pm .5$ second perturbation in the ignition time. So within the $\pm .5$ second range about its nominal value, the variations in this parameter are not vital to the success of the mission.

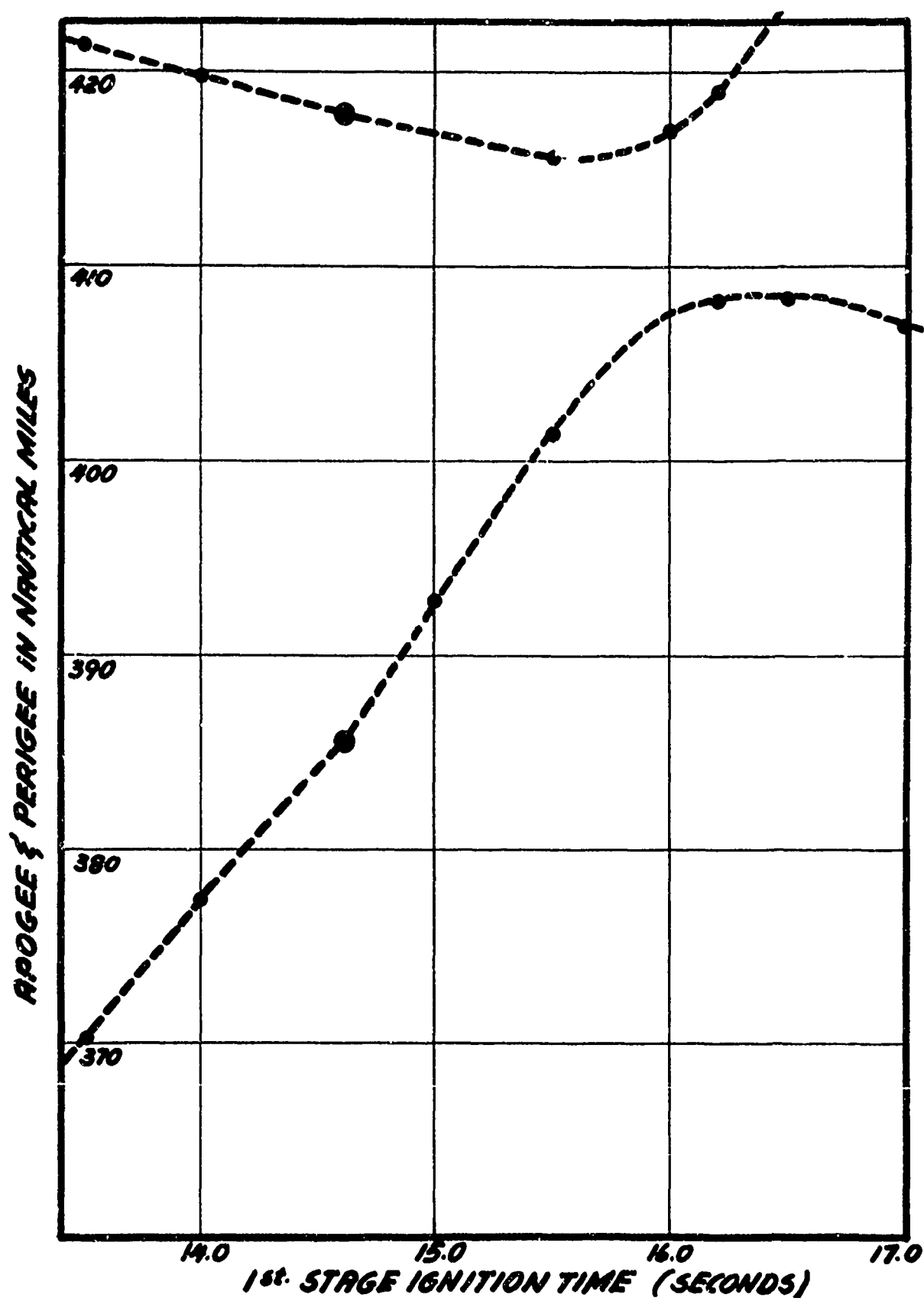


FIGURE 3.2.1

TABLE 3.2.1

1st stage Ignition Time (P 2.1) Cases 3059-3062 and 3133-3136

1st Ignition time	2nd I.T. after 2nd stg BO	3rd I.T. after 2nd stg BO	Vehicle Attitude at 1st stg. B.O.		At 2nd stage ignition			Orbit Parameters	
			Elev.	Azim.	Height	Abs. Vel.	Path Angle	Perigee	Apogee
second	second	second	degree	degree	N.M.	ft/sec	degree	N.M.	N.M.
13.5	1.61	564.4	27.49	3.33	17.24	13 487	24.72	370.18	421.26
14.0	1.11	564.4	27.33	3.33	16.91	13 506	24.62	377.44	419.78
14.61	0.5	564.4	27.13	3.33	16.51	13 529	24.50	385.52	417.77
15.00	0.5	564.0	27.00	3.32	16.62	13 530	24.38	392.72	416.70
15.5	0.5	563.5	26.83	3.32	16.75	13 531	24.23	401.41	415.66
16.0	0.5	563.0	26.67	3.32	16.88	13 531	23.11	407.57	416.91
16.2	0.5	562.8	26.60	3.31	16.94	13 531	24.02	408.42	418.95
16.5	0.5	562.5	26.50	3.31	17.01	13 531	23.93	408.34	423.28
17.0	0.5	562.0	26.33	3.31	17.14	13 531	23.77	406.87	431.64

3.2.2 First Stage Fuel Specific Impulse (P 2,4)

The results appear in Table 3.2.2 and Figure 3.2.2. The Specific Impulse of the propellant has a definite effect on the orbit parameters. Below the nominal value of 280 seconds, the perigee decreases at an average rate of 16 nautical miles per second. Above 285 seconds, the perigee remains approximately constant at 407 nautical miles while the apogee increases at an average rate of 16 nautical miles per second of specific impulse. To more specifically evaluate the importance of this parameter, more information would be needed about the rocket motors; when this information will be available, it will be easy to determine the exact role of this parameter.

3.2.3 First Stage Burning Rate (P 2,5)

Note here that the amount of fuel in the first stage is constant at 1619 pounds; only the burning rate was changed. The results appear in Figure 3.2.3 and Table 3.2.3. There is an almost not noticeable discontinuity in the rate of change of the orbit parameters which is easily explained by the fact that when the authors tried to reduce the burning rate, there was not enough time left prior to second stage ignition to do it; it was then necessary to delay the second stage ignition since the half a second between the first stage burnout and the second stage ignition was not sufficient.

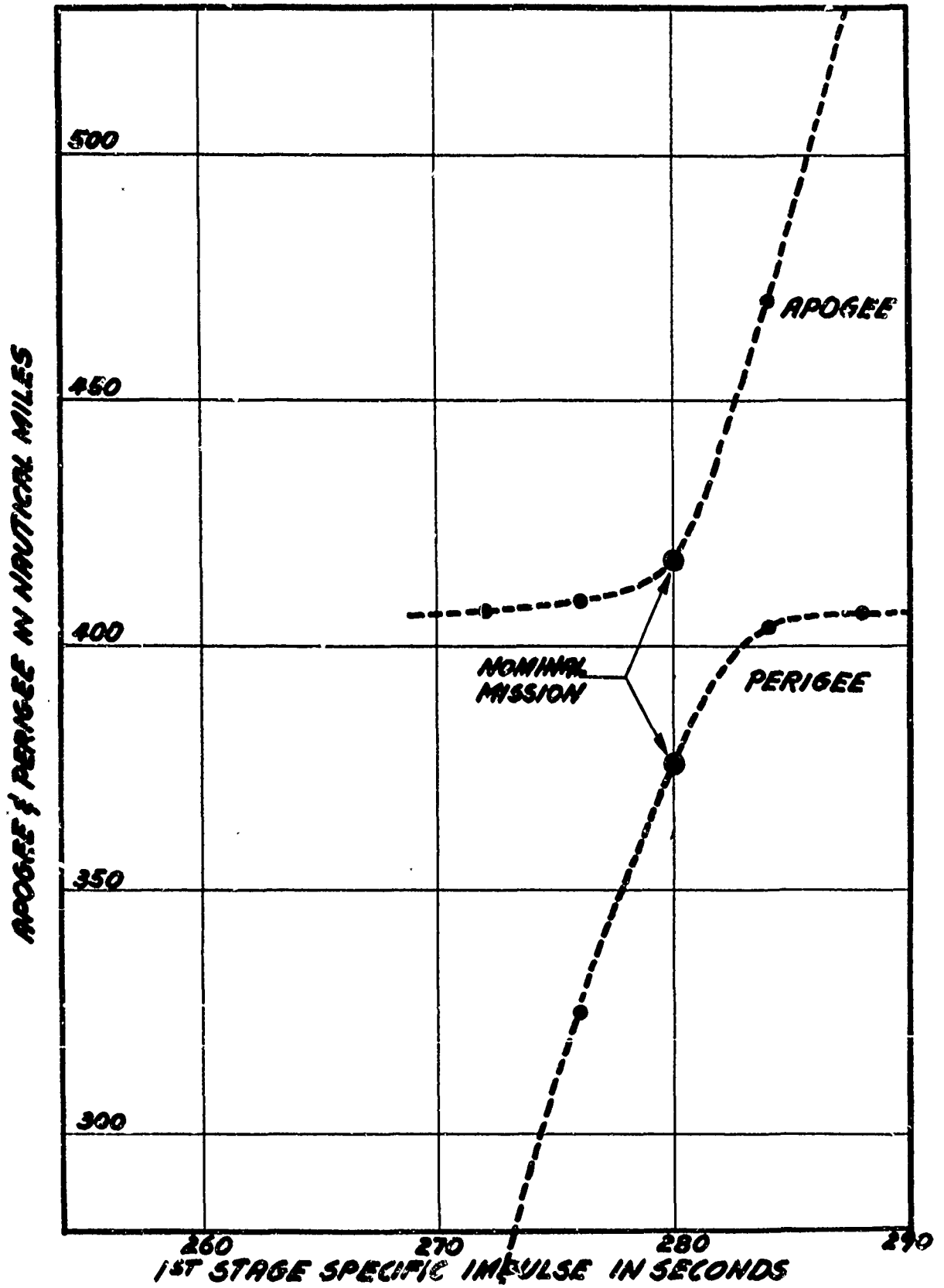
**FIGURE 3.2.2**

TABLE 3.2.2
1st Stage Specific Impulse (P 2.4) Cases 3055 - 3058

Specific Impulse	Vehicle Attitude 1st stg BO		At 2nd Stage Ignition			Orbit Parameters	
	Elevation	Azimuth	Height	Abs. Vel.	Path Angle	Perigee	Apogee
	degree	degree	N.M.	ft/sec	degree	N.M.	N.M.
second							
272	27.09	3.38	16.39	13 302	24.42	258.92	407.03
276	27.11	3.35	16.45	13 415	24.46	324.87	409.00
280	27.13	3.33	16.51	13 529	24.50	385.52	417.77
284	27.15	3.30	16.57	13 642	24.54	403.81	470.45
288	27.16	3.27	16.64	13 756	24.57	406.43	540.41

Figure 3.2.3 indicates that a slower burning rate would be recommended to get a higher perigee; the apogee is not very sensitive to this parameter.

As a consequence, it would be interesting to study the orbits obtained with burning rates below 100 pound per second; this would also help to optimize in a better way the nominal burning rate. However the fact that a slower burning rate necessitates a delay in the second stage ignition time makes the two parameters correlated and any optimization should be carried out with respect to these two parameters varied simultaneously.

Here again no conclusions can be drawn in the absence of more information on the rocket motor.

3.2.4 First Stage Weight of Fuel (P 2,6)

This parameter was varied keeping constant the fuel burning rate and the structural weight of the first stage. The results appear in Table 3.2.4 and Figure 3.2.4. As in the previous first stage parameters we obtain an hyperbolic behaviour with two assymptotes: below the nominal 1619 pounds of fuel, the apogee remains almost constant at 405 nautical miles while the perigee decreases at a rate of approximately 2.2 nautical miles per pound of fuel. Similarly with more than 1640 pounds of fuel, the perigee keeps a constant value at 405 nautical miles while the apogee increases at a rate

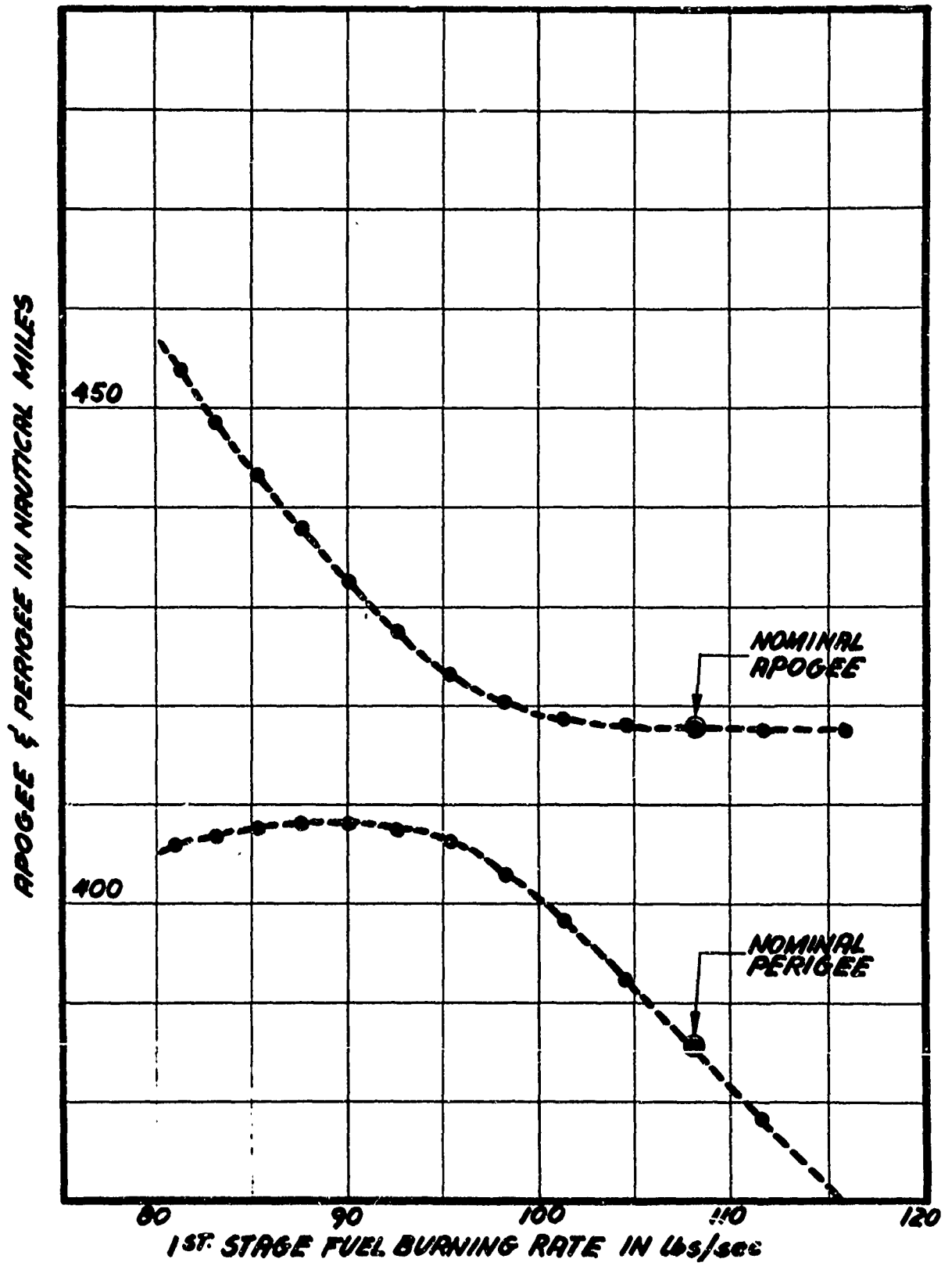


FIGURE 3.2.3

TABLE 3.2.3

1st Stage Fuel Burning Rate (P 2.5)

1st Stage Burning Rate	2nd Ignition Time	3rd Ignition Time	Vehicle Attitude			At 2nd Stage Ignition			Orbit Parameters	
			At 1st stg. BO		Azim.	Height	Abs. Vel.	Path Angle	Perigee	Apogee
			Elev.	degree						
lbs/sec	sec	sec	degree	degree	degree	N.M.	ft/sec	degree	N.M.	N.M.
115.64	30.11	604.51	27.32	3.33	3.33	16.87	13 502	24.58	376.83	417.94
111.63	30.11	604.51	27.23	3.33	3.33	16.69	13 516	24.54	381.36	417.88
107.93	30.11	604.51	27.13	3.33	3.33	16.51	13 529	24.50	385.52	417.77
104.45	30.61	604.51	27.03	3.33	3.33	16.79	13 524	24.41	392.33	418.05
101.18	31.11	604.51	26.93	3.32	3.32	17.07	13 520	24.32	398.30	418.78
98.12	31.61	604.51	26.83	3.32	3.32	17.35	13 510	24.22	403.06	420.38
95.23	32.11	604.51	26.73	3.32	3.32	17.62	13 511	24.13	406.18	423.28
92.51	32.61	604.51	26.64	3.32	3.32	17.90	13 507	24.04	407.67	427.51
89.94	33.11	604.51	26.54	3.32	3.32	18.17	13 503	23.95	408.13	432.50
87.51	33.61	604.51	26.44	3.32	3.32	18.44	13 499	23.86	408.00	437.80
85.21	34.11	604.51	26.34	3.32	3.32	18.71	13 495	23.77	407.53	443.20
83.02	34.61	604.51	26.24	3.32	3.32	18.97	13 490	23.68	406.83	448.57
80.95	35.11	604.51	26.14	3.32	3.32	19.24	13 486	23.58	405.99	453.86

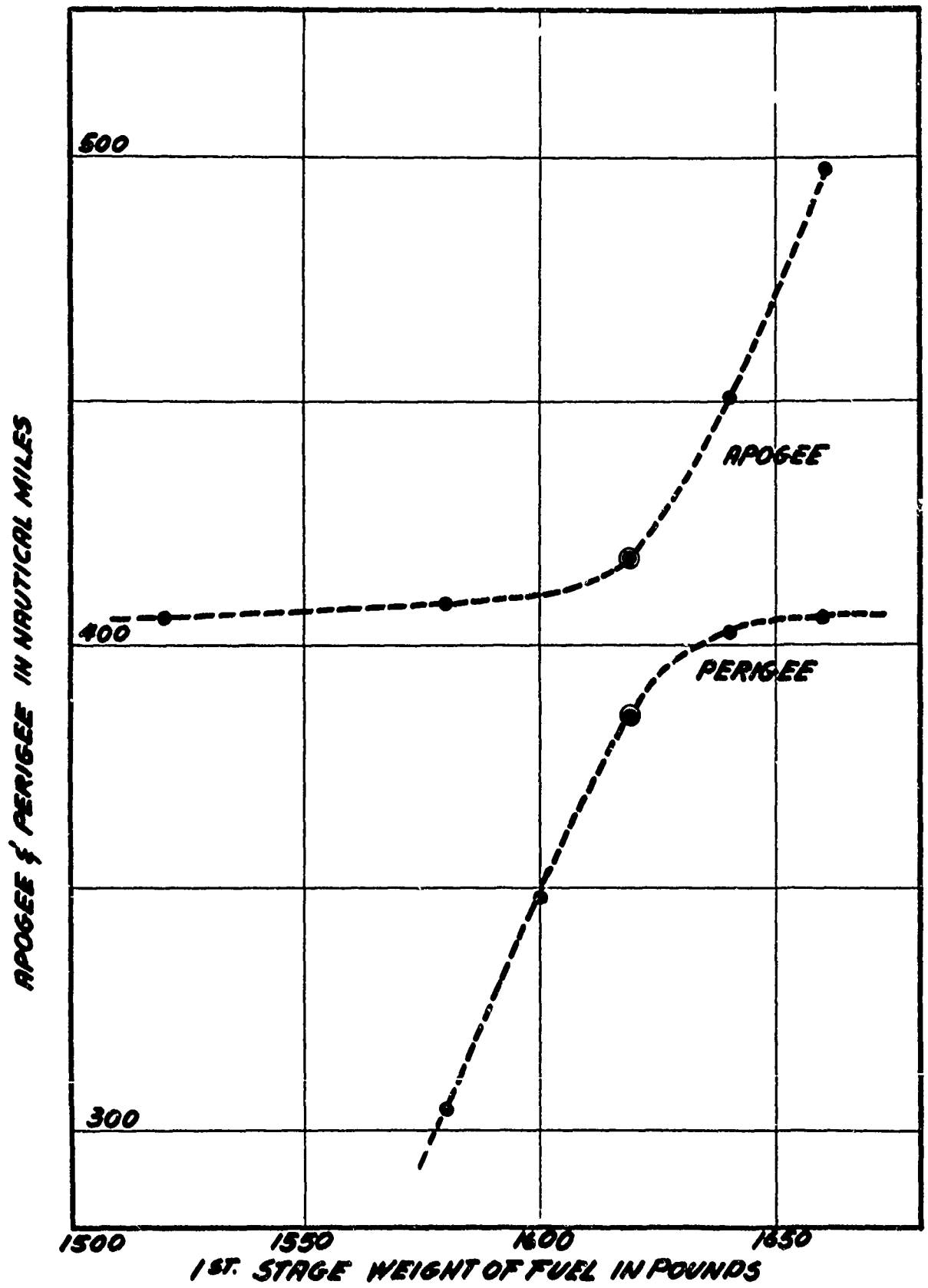


FIGURE 3.2.4

TABLE 3.2.4
1st Stage Fuel Weight (P 2.6) Cases 3067 - 3071

1st stage Weight of Fuel	Vehicle Attitude 1st stg. BO		At 2nd Stage Ignition			Orbit Parameters	
	Elevation	Azimuth	Height	Abs. Vel.	Path Angle	Perigee	Apogee
	degree	degree	N.M.	ft/sec	degree	N.M.	N.M.
lbs							
1520	27.23	3.41	16.59	13 134	24.43	168.80	405.55
1580	27.17	3.36	16.55	13 376	24.47	304.32	408.40
1600	27.15	3.34	16.53	13 454	24.49	348.28	410.59
1619	27.13	3.33	16.51	13 529	24.50	385.52	417.77
1640	27.11	3.31	16.67	13 603	24.49	402.53	450.99
1660	27.09	3.29	16.82	13 675	24.49	405.74	497.26

of 2.2 nautical miles per pound of fuel. Consequently a variation of one pound in the weight of fuel only produces a 2.2 nautical miles variation in the perigee of the nominal orbit; hence this parameter is apparently not crucial for the success of the mission.

3.3 PHASE 3 - Second Stage

Phase 3 of a Martlet IV trajectory is defined by a series of parameters at 2nd stage ignition. These are:

- a) P 3.1 - The time after first stage burnout when second stage ignition takes place.
- b) P 3.2 - Elevation or pitch angle of rocket.
- c) P 3.3 - Azimuth or yaw angle of rocket.
- d) P 3.4 - Specific Impulse of 2nd stage propellant.
- e) P 3.5 - Burning rate of fuel or equivalently the fuel burning time for a given propellant weight.
- f) P 3.6 - Weight of fuel.

Each of these parameters is independent of the rest and of that portion of the trajectory which precedes phase 3.

3.3.1 Second Stage - Ignition Time (P 3.1)

Dispersion in apogee and perigee due to errors in the ignition time of the second stage is shown in graph 3.3.1. The abscissa is scaled with the number of seconds after first stage burnout which occurs at 29.61 seconds after gun launch.

As shown positive errors do not endanger the orbital capability of the mission but it has an improving effect in that it makes the orbit more circular with a higher perigee. The kind of errors or dispersion expected in this parameter is not known but provided it is within the range shown in the graph that is ± 9 seconds, orbit will be achieved. It is expected that for sufficiently high ignition times a critical maximum value will be reached.

This study is encouraging as far as the possibility for correcting tip-off errors is concerned. It shows that an attitude control system could have an interval of 10 seconds in which to sense and correct attitude errors due to tip-off disturbances.

3.3.2 Second Stage - Firing Elevation or Pitch (P 3.2)

The effect of dispersion in firing elevation on the apogee and perigee is shown in graph 3.3.2. The dispersion will depend on whether an attitude control system is present or not. If a guidance system is present it can be expected to orient the vehicle within ± 2 degrees provided it has enough time to do it. Since the interval of time between first stage burnout and second stage ignition is of the order of a few seconds, reorientation may not be possible if a large correction is needed. Such large errors are possible when tip-off occurs during first stage separation, therefore we have looked at a

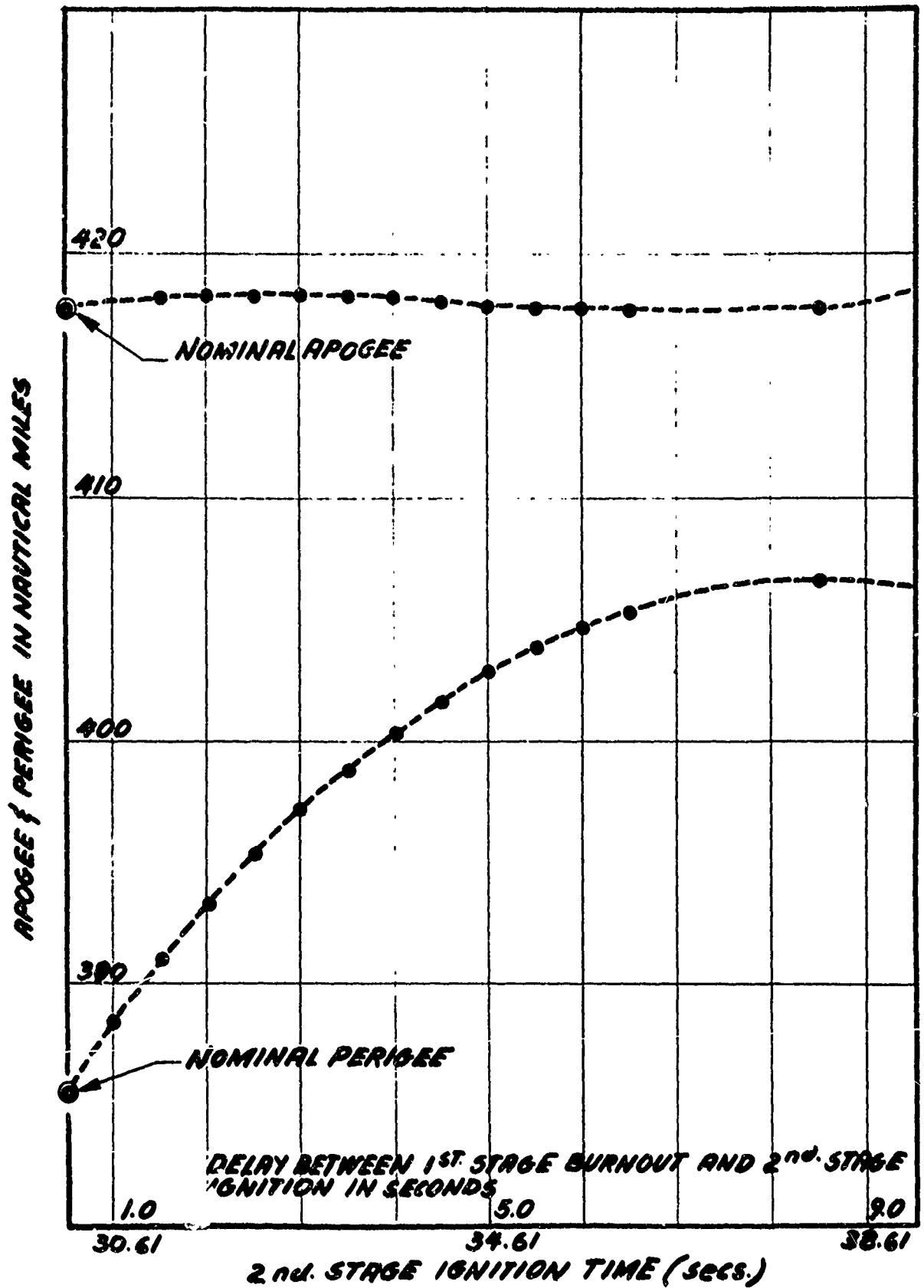
**FIGURE 3.3.1**

TABLE 3.3.1

2nd Stage Ignition Time

2nd Stage Ignition Time	2nd Stage B.O. Vehicle Attitude		3rd Stage Ignition			Orbit Parameters	
	Elev.	Azim.	Height	Abs. Vel.	Path Angle	Perigee	Apogee
seconds	degree	degree	N.M.	ft/sec	degree	N.M.	N.M.
30.11	27.48	2.26	415.76	16,313.	0.26	385.52	417.77
30.61	27.48	2.26	415.44	16,319.	0.27	388.44	418.04
31.11	27.48	2.26	415.28	16,325.	0.27	391.02	418.21
31.61	27.48	2.26	414.95	16,331.	0.27	393.32	418.32
32.11	27.48	2.26	414.58	16,337.	0.27	395.35	418.36
32.61	27.48	2.25	414.16	16,343.	0.27	397.20	418.33
33.11	27.48	2.25	413.72	16,348.	0.27	398.83	418.29
33.61	27.48	2.25	413.24	16,353.	0.26	400.30	418.22
34.11	27.48	2.25	412.74	16,358.	0.26	401.63	418.09
34.61	27.48	2.25	412.22	16,363.	0.25	402.82	417.93
35.11	27.48	2.25	411.68	16,368.	0.24	403.81	417.77
35.61	27.48	2.25	411.12	16,373.	0.23	404.66	417.64
36.11	27.48	2.25	410.55	16,377.	0.22	405.38	417.54
38.11	27.48	2.24	408.16	16,395.	0.17	406.64	417.84
39.11	27.48	2.24	406.91	16,403.	0.14	406.34	418.43

deviation in pitch of ± 4 degrees from its nominal value of 27.08 degrees. If a control system is not present, then one has to allow for the presence of large tip-off errors and graph 3.3.2 indicates the behaviour of the orbit under such conditions. From this graph it can be seen that pitch errors larger than $\pm 4^\circ$ can be found by linear extrapolation of the apogee and perigee curves. The critical pitch angles, that is those which yield a perigee of 100 N.M., can be found from such an extrapolation to be 34.7 degrees and 8 degrees in the positive and negative directions respectively. Thus errors of $+ 7.7^\circ$ and $- 19^\circ$ about the nominal pitch angle can occur and the perigee higher than 100 N.M. will still be obtained. Finally, one can say that the perigee is very sensitive to this parameter, particularly to positive errors, and that a more circular orbit can be achieved for a nominal pitch of approximately 26.5° .

3.3.3 Second Stage - Firing Azimuth (P 3.3)

The effect of dispersion in firing azimuth is shown in graph 3.3.3. Similar remarks concerning possible dispersion due to tip-off or otherwise, apply here as those in the elevation (Section 3.3.2). The orbit sensitivity to this parameter is not as critical as in the elevation case, positive errors yielding slightly more dispersion than negative ones.

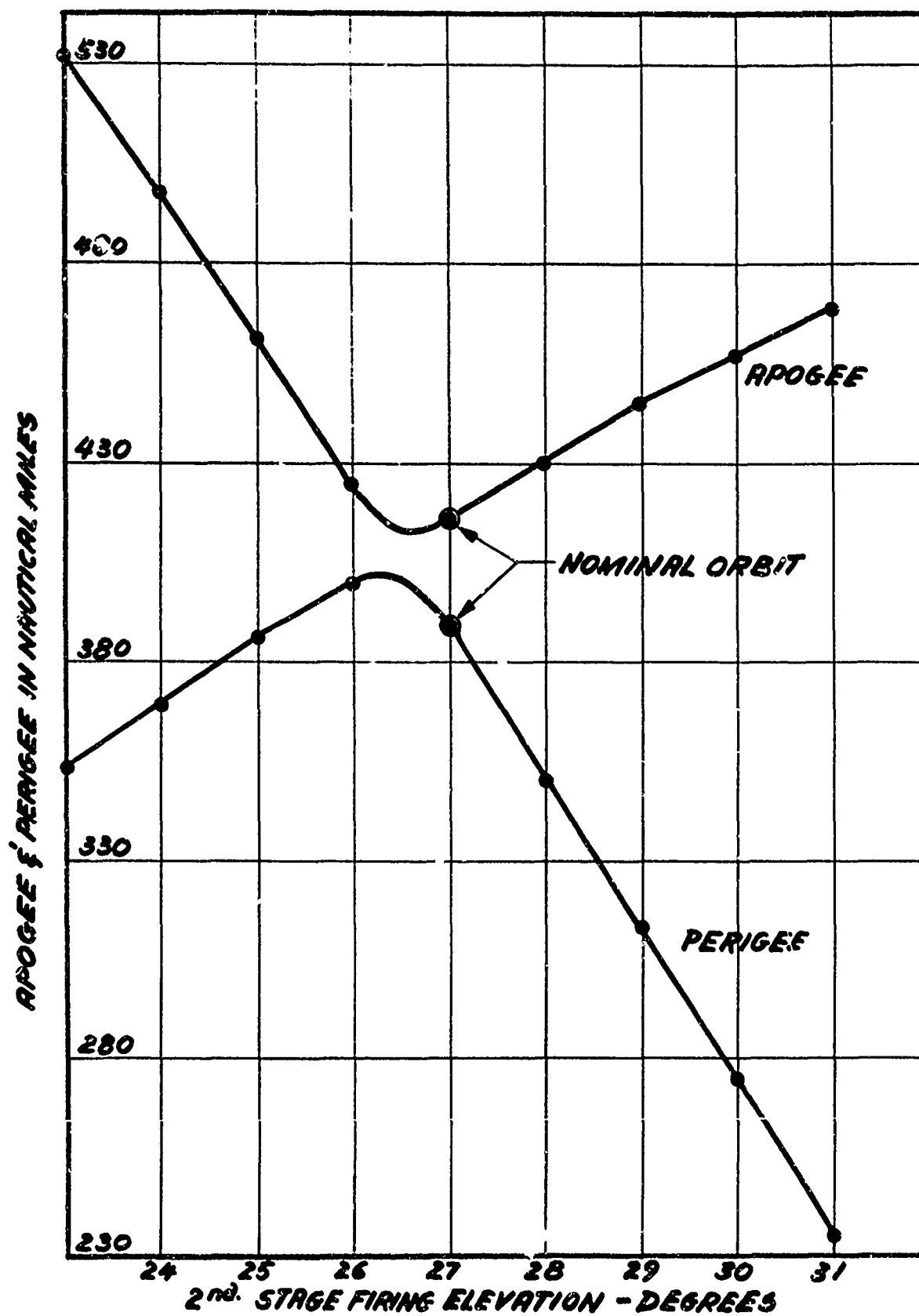
**FIGURE 3.3.2**

TABLE 3.3.2
2nd Stage Firing Elevation Cases 3088 - 3096

2nd Stg. Firing Elev.	2nd Stg. F.O. Vehicle Attitude		3rd Stage Ignition			Orbit Parameters	
	Elev. degree	Azim. degree	Height N.M.	Abs. Vel. ft/sec	Path Angle degrees	Perigee N.M.	Apogee N.M.
23.0	23.41	2.23	373.14	16,718.	-1.12	354.34	532.91
24.0	24.41	2.23	383.94	16,619.	-0.77	370.75	497.66
25.0	25.41	2.24	394.52	16,520.	-0.43	386.56	461.76
26.0	26.40	2.25	404.86	16,420.	-0.09	401.74	425.58
27.0	27.40	2.26	414.96	16,321.	0.23	388.47	416.67
28.0	28.40	2.27	424.83	16,221.	0.55	351.17	420.68
29.0	29.40	2.28	434.46	16,121.	0.86	313.35	444.23
30.0	30.40	2.28	443.86	16,020.	1.16	275.07	457.22
31.0	31.40	2.29	453.32	15,919.	1.46	236.37	469.69

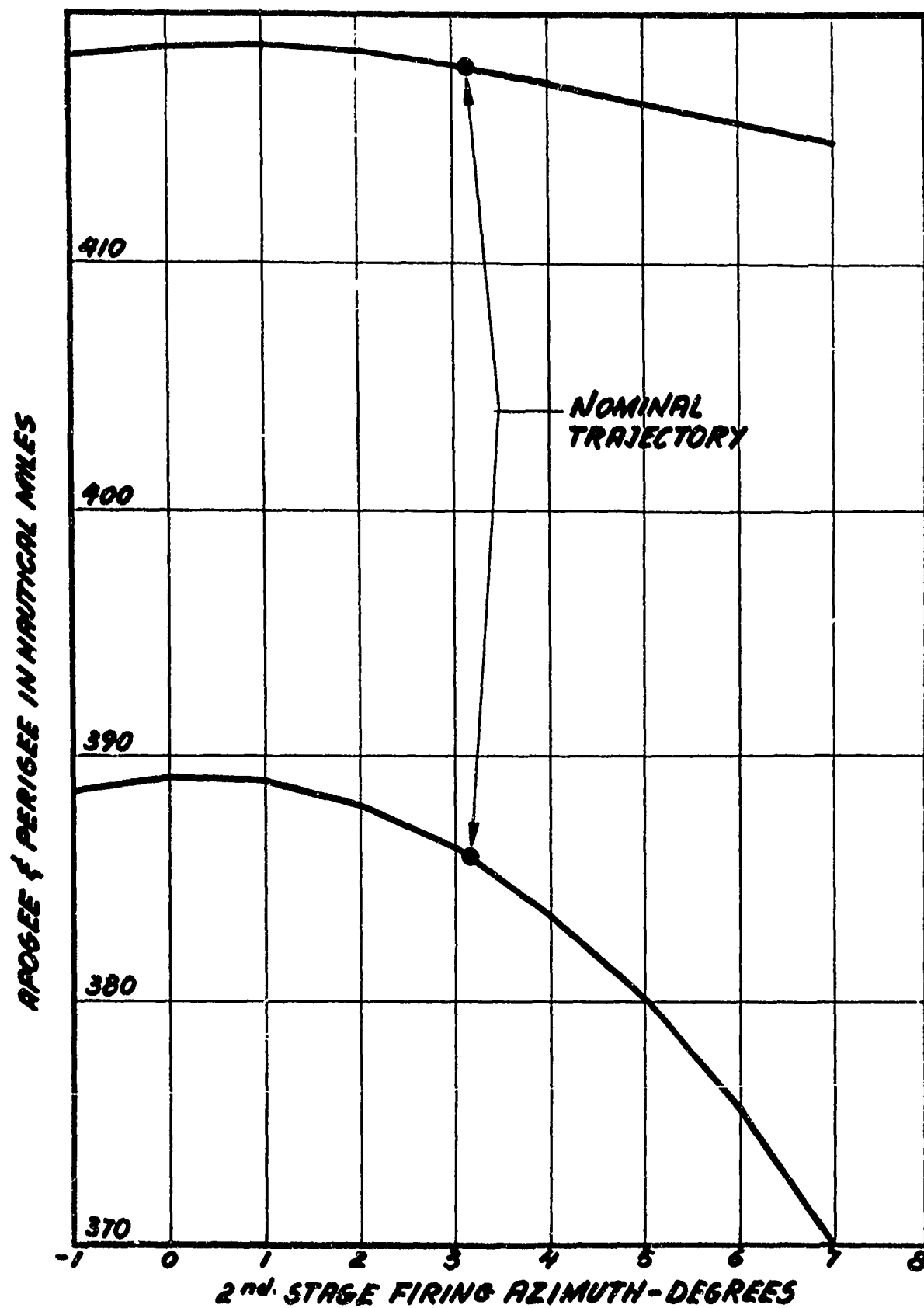


FIGURE 3.3.3

TABLE 3,3.3

2nd Stage Firing Azimuth Cases 3097 - 3105

2nd Stage Firing Az.	2nd Stage B. O. Vehicle Attitude		3rd Stage Ignition			Orbit Parameters	
	Elev.	Azim.	Height	Abs. Vel.	Path Angle	Perigee	Apogee
degrees	degrees	degrees	N.M.	ft/sec	degrees	N.M.	N.M.
-1.0	27.48	-0.33	415.86	16,318.	0.27	388.56	418.40
0.0	27.48	0.27	415.98	16,319.	0.28	389.10	418.75
1.0	27.48	0.87	416.01	16,318.	0.28	388.92	418.78
2.0	27.48	1.46	415.96	16,317.	0.27	387.99	418.51
3.0	27.48	2.06	415.82	16,314.	0.26	386.26	418.00
4.0	27.48	2.66	415.60	16,310.	0.25	383.69	417.28
5.0	27.48	3.26	415.30	16,305.	0.23	380.21	416.44
6.0	27.48	3.86	414.92	16,229.	0.20	375.70	415.60
7.0	27.48	4.45	414.45	16,292.	0.17	370.15	414.78

3.3.4 Second Stage - Specific Impulse (P 3.4)

The effect of variations in propellant specific impulse in the second stage is shown in graph 3.3.4. An arbitrary variation of ± 10 secs. about the nominal value of 290 secs. was chosen. Actual possible dispersion was not available but it is clear from the graph that the perigee will fall to 100 N.M. value if the specific impulse should drop below a critical minimum. This critical value is approximately given by:

$$P\ 3.4 = 269\ \text{secs.}$$

Thus one can conclude that all other errors remaining at zero, provided the specific impulse is above 269, orbit will be achieved. Deviations in the positive direction quite clearly will not impede an orbit from being achieved.

3.3.5 2nd Stage - Burning Rate (P 3.5)

The burning rate of the propellant is found by dividing the amount of propellant by the burning time. This quantity is not constant throughout the burning time, therefore, an average value is found. The size of the possible error is not known; however, for the burning rate in the 12 to 44 lbs per second range, the rate of change of the perigee is -0.57 nautical mile per pound per second. The results are shown in graph 3.3.5. The sensitivity of the apogee is very small to this parameter in the estimated range as it varies by ± 2 N.M. One can conclude that this parameter is not crucial to the success of the nominal mission.

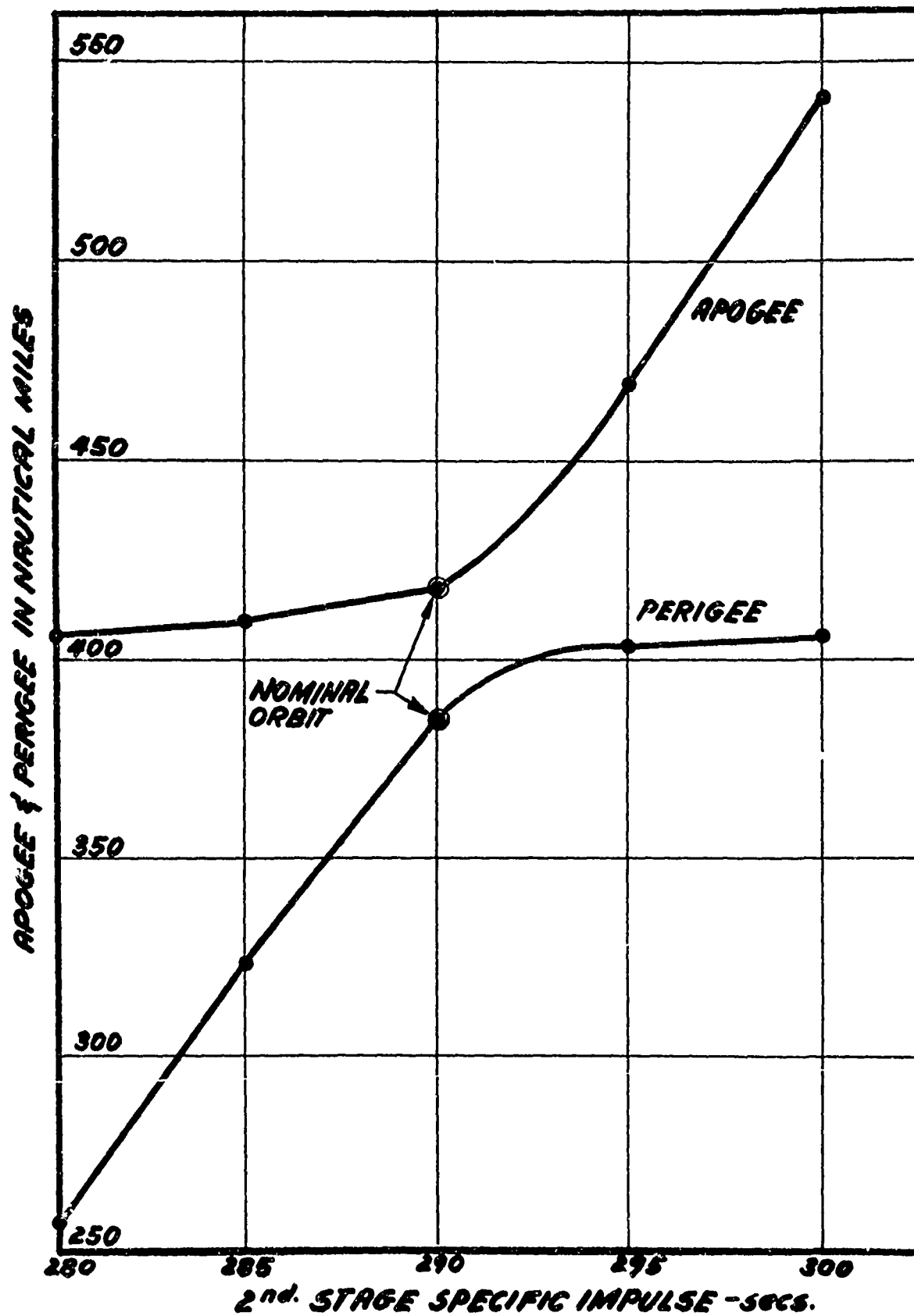
**FIGURE 3.3.4**

TABLE 3.3.4

2nd Stage Specific Impulse Cases 3072 - 3075

2nd Stage Sp. Imp.	2nd Stg. B.O. Vehicle Attitude		3rd Stage Ignition			Orbit Parameters	
	Elev.	Azim.	Height	Abs. Vel.	Path Angle	Perigee	Apogee
secs	deg	deg	N.M.	ft/sec	deg	N.M.	N.M.
280	27.48	2.28	397.12	16,207.	-0.77	257.82	407.21
285	27.48	2.27	406.51	16,259.	-0.25	324.20	409.19
290	27.48	2.26	415.76	16,313.	0.26	385.52	417.77
295	27.49	2.25	425.10	16,367.	0.77	404.17	470.61
300	27.49	2.23	434.44	16,423.	1.27	406.82	541.08

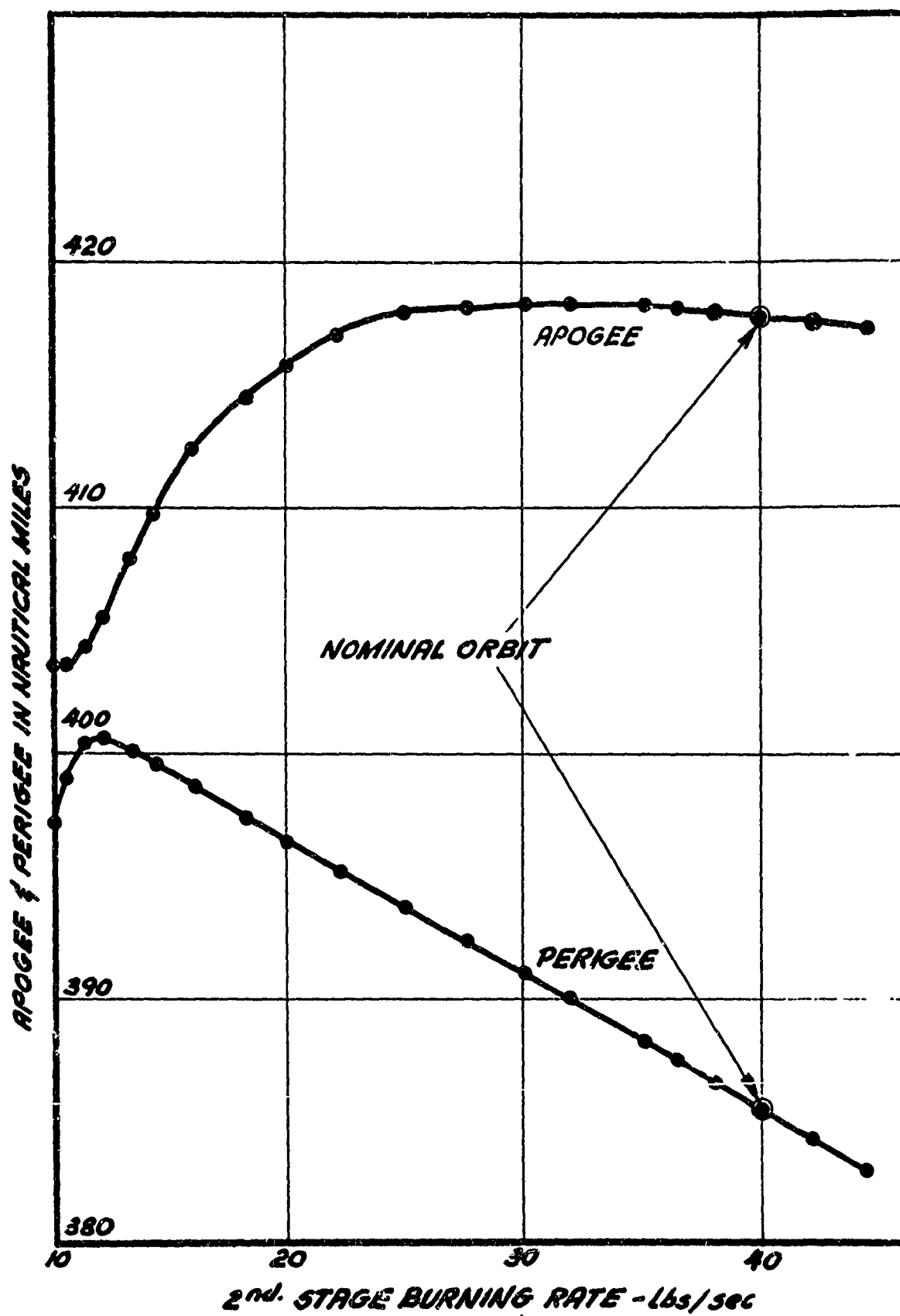
**FIGURE 3.3.5**

TABLE 3.3.5

2nd Stage Burning Rate

2nd Stage Burning Rate	2nd Stage B.O. Vehicle Attitude		3rd Stage Ignition			Orbit Parameters	
	Elev.	Azim.	Height	Abs. Vel.	Path Angle	Perigee	Apogee
lb/sec	degree	degree	N.M.	ft/sec	degree	N.M.	N.M.
44.4	27.44	2.26	415.79	16,308.	0.24	383.00	417.32
42.1	27.46	2.26	415.79	16,310.	0.25	384.31	417.57
40.0	27.46	2.26	415.76	16,313.	0.26	385.52	417.77
38.1	27.50	2.26	415.71	16,315.	0.27	386.62	416.96
36.4	27.52	2.26	415.64	16,318.	0.27	387.61	418.10
35.1	27.54	2.26	415.58	16,319.	0.27	388.33	418.21
32.0	27.58	2.26	415.34	16,324.	0.28	390.08	418.37
30.1	27.62	2.26	415.12	16,327.	0.28	391.15	418.42
27.6	27.66	2.26	414.72	16,331.	0.28	392.43	418.28
25.0	27.72	2.26	414.17	16,337.	0.28	393.78	417.86
22.2	27.80	2.26	413.27	16,343.	0.27	395.21	417.01
20.0	27.88	2.27	412.32	16,349.	0.25	396.38	415.91
18.2	27.96	2.27	411.30	16,355.	0.23	397.87	414.56
16.0	28.08	2.27	409.71	16,363.	0.20	398.56	412.30
14.3	28.20	2.27	408.05	16,372.	0.17	399.56	409.78
13.3	28.28	2.27	406.98	16,377.	0.15	400.11	408.05
12.1	28.40	2.28	405.22	16,385.	0.11	400.64	405.57
11.4	28.48	2.28	404.07	16,390.	0.08	400.53	404.28
10.5	28.60	2.28	402.83	16,397.	0.04	399.04	403.55
10.0	28.68	2.28	401.17	16,402.	0.02	397.31	403.74

3.3.6 Second stage - Weight of Fuel (P 3.6)

This parameter was varied keeping constant the fuel burning rate and the structural weight of the second stage. The results appear in Table 3.3.6 and Figure 3.3.6. The apogee and perigee vary asymptotically with the second stage weight of fuel, w_2 :

$$w_2 \leq 400 \text{ pounds,}$$

$$\text{Apogee} \longrightarrow 408 \text{ N.M.}$$

$$\text{Perigee} = 386 + 5.15 (w_2 - 400) \text{ N.M.}$$

$$w_2 \geq 410 \text{ pounds,}$$

$$\text{Apogee} = 446 + 6.3 (w_2 - 410) \text{ N.M.}$$

$$\text{Perigee} \longrightarrow 406 \text{ N.M.}$$

For a variation of one pound of fuel about the nominal 400 pounds of fuel, the corresponding changes in the orbit parameters are:

$$\Delta \text{ apogee} = \begin{matrix} +3.0 \\ -1.0 \end{matrix} \text{ N.M. (nominal : 417.0 N.M.)}$$

$$\Delta \text{ perigee} = \begin{matrix} +5.5 \\ -4.0 \end{matrix} \text{ N.M. (nominal : 385.5 N.M.)}$$

It then seems that this parameter is not vital to the success of the mission.

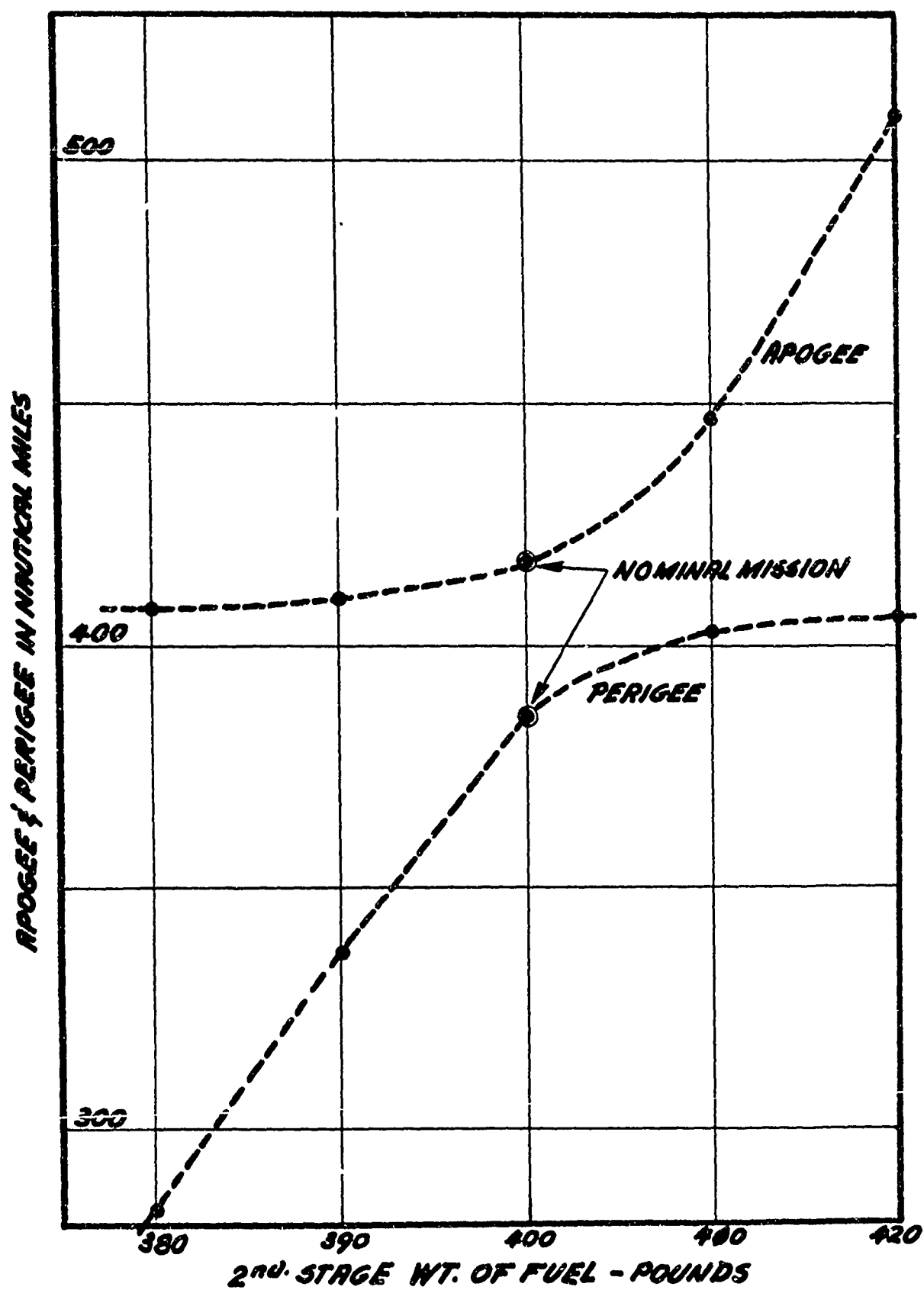


FIGURE 3.3.6

TABLE 3.3.6

Second stage Weight of Fuel (P 3,6) Cases 3084 - 3087

2nd stage Weight of Fuel	At 3rd stage Ignition Time			Orbit Parameters	
	Height	Abs. Vel.	Path Angle	Perigee	Apogee
	N.M.	ft/sec	degree	N.M.	N.M.
pounds					
380	401.08	16 224	-0.56	282.82	408.01
390	408.51	16 269	-0.15	337.18	410.01
400	415.55	16 319	+0.27	385.52	417.17
410	422.92	16 357	+0.66	402.90	456.86
420	429.89	16 401	+1.04	406.06	509.61

3.4 PHASE 4 - 3rd Stage

Phase 4 is defined by the following parameters at third stage ignition:

- a) P 4.1 - The time after second stage burnout when third stage ignition occurs.
- b) P 4.2 - The elevation or pitch angle of the vehicle.
- c) P 4.3 - Azimuth or yaw angle of the vehicle.
- d) P 4.4 - Specific impulse of third stage propellant.
- e) P 4.5 - Burning rate of fuel or equivalently the burning time for a given fuel weight.
- f) P 4.6 - Weight of fuel.

Each of these parameters is independent of the other and of that part of the trajectory which precedes phase 4.

3.4.1 Third stage - Ignition Time (P 4.1)

The results appear in Table 3.4.1 and Figure 3.4.1. If the ignition time had been delayed by 2.5 seconds a very slightly better orbit would have been obtained.

To a variation of half a second in the nominal ignition time of 604.51 seconds, correspond the following maximum possible variations in the apogee and perigee:

perigee = ± 0.5 N.M. (nominal 385.5 N.M.)

apogee = ± 0.5 N.M. (nominal 417.8 N.M.)

The above variations are inferior to a change of one per cent and consequently this parameter does not have any significant effect on the success of the mission if it is compared to other parameters.

3.4.2 3rd Stage - Firing Elevation (P 4.2)

The results of dispersion in this parameter are shown in graph 3.4.2. An attitude control system is essential to provide a nominal pitch angle of zero degrees before 3rd stage ignition in order to ensure orbit. Extrapolation of the curve indicates that a tolerance of ± 4 degrees would keep perigee above 300 N.M. and ± 13 degrees would keep perigee greater than 100 N.M. If the attitude control system is then jettisoned before ignition the total pitch error, including the effects of tip-off caused by jettisoning the control package, would need to be less than ± 2 degrees in order to keep the perigee above 350 N.M.

3.4.3 3rd Stage - Firing Azimuth

The results are shown in graph 3.4.3. The expected dispersion with a control system is $\pm 1^\circ$, however, if the ejection of this system is done prior to third stage firing then tip-off errors will become a factor. The sensitivity to this error is so low that one can ignore its effect in the shown range.

These results seems to indicate that the azimuth need not be very strictly controlled, and this should be investigated in more detail. The effect of such a simplification could greatly reduce the complexity of the guidance system.

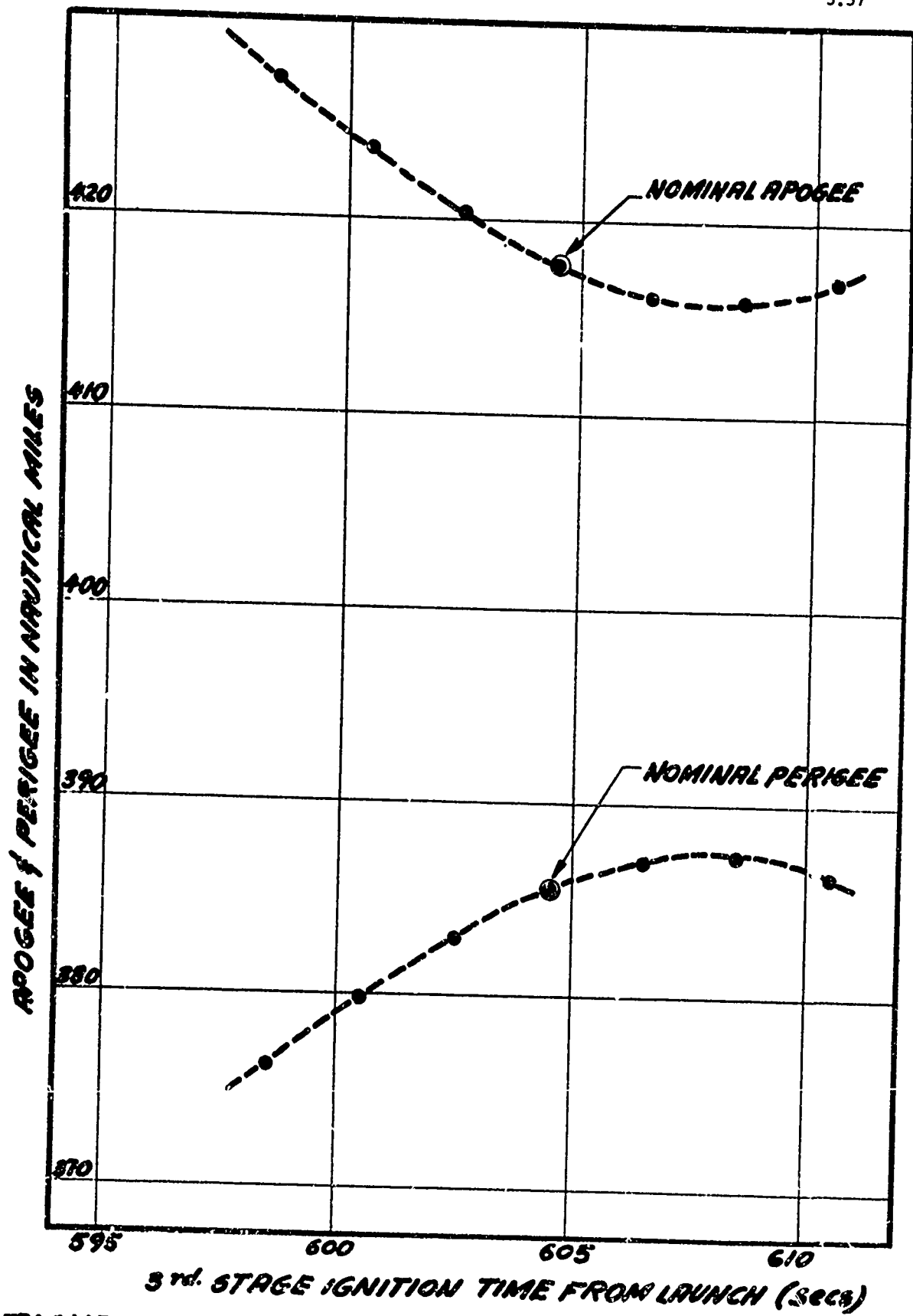


FIGURE 3.4.1

TABLE 3.4.1Third stage Ignition Time (P 4.1) Cases 3142 - 3148

3rd Stage Ignition Time	Orbit Parameters	
	Perigee	Apogee
second	N.M.	N.M.
598.51	376.30	427.09
600.51	379.83	423.51
602.51	382.99	420.33
604.51	385.52	417.77
606.51	387.09	416.18
608.51	387.39	415.87
610.51	386.36	416.90

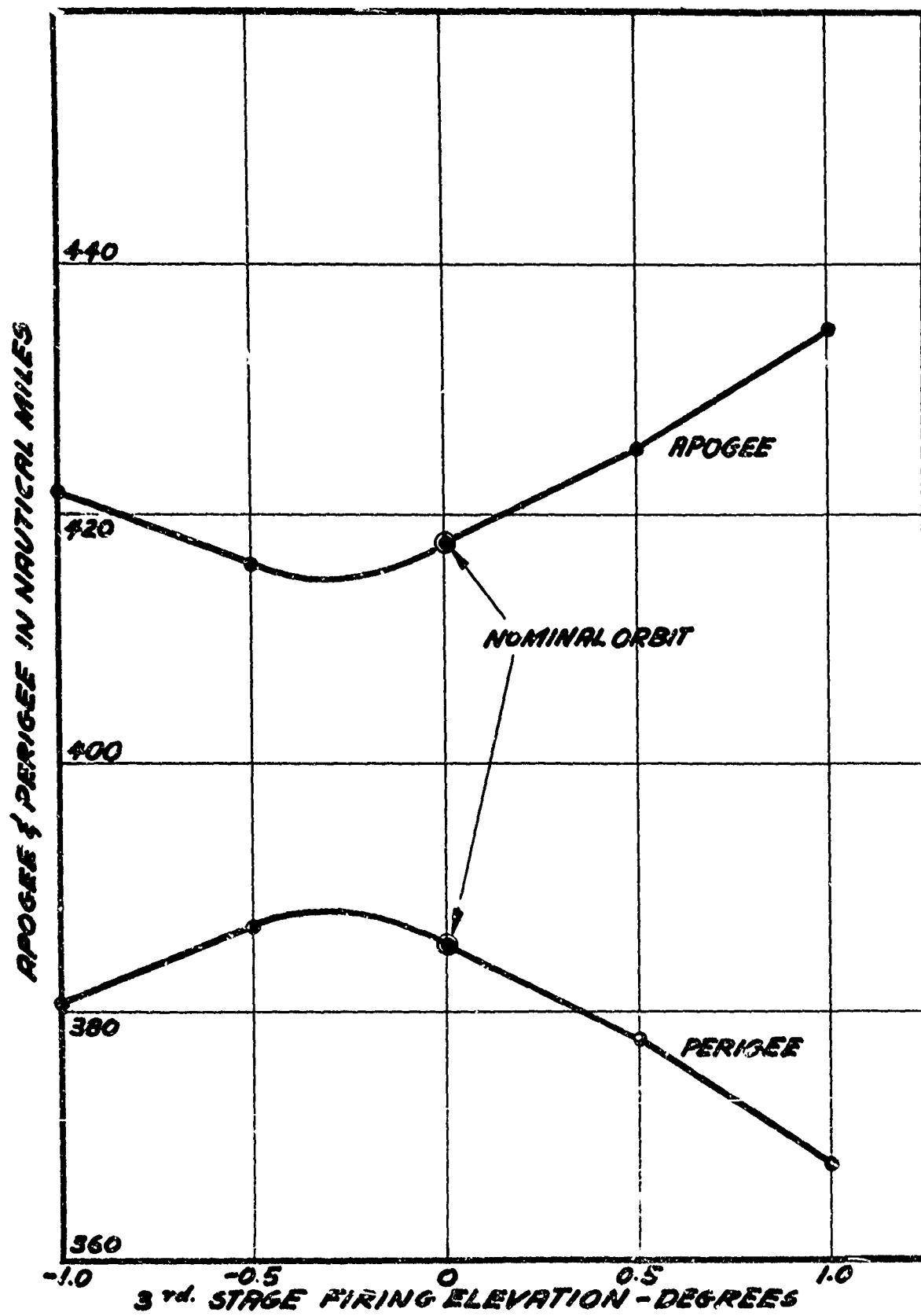


FIGURE 3.4.2

TABLE 3.4.2
3rd Stage Firing Elevation Cases 3123 - 3126

3rd Stage Firing Elevation	3rd Stage Ignition		3rd Stage B. O.			Orbit Parameters	
	Height	Path Angle	Height	Abs. Vel.	Path Angle	Perigee	Apogee
	N.M.	degrees	N.M.	ft/sec	degrees	N.M.	N.M.
-1.0	415.76	0.26	415.75	24,454.	-0.22	380.83	421.94
-0.5	415.76	0.26	415.78	24,454.	-0.05	386.93	416.23
0.0	415.76	0.26	415.80	24,454.	0.11	385.52	417.77
0.5	415.76	0.26	415.83	24,454.	0.28	377.94	425.22
1.0	415.76	0.26	415.85	24,453.	0.45	368.05	434.73

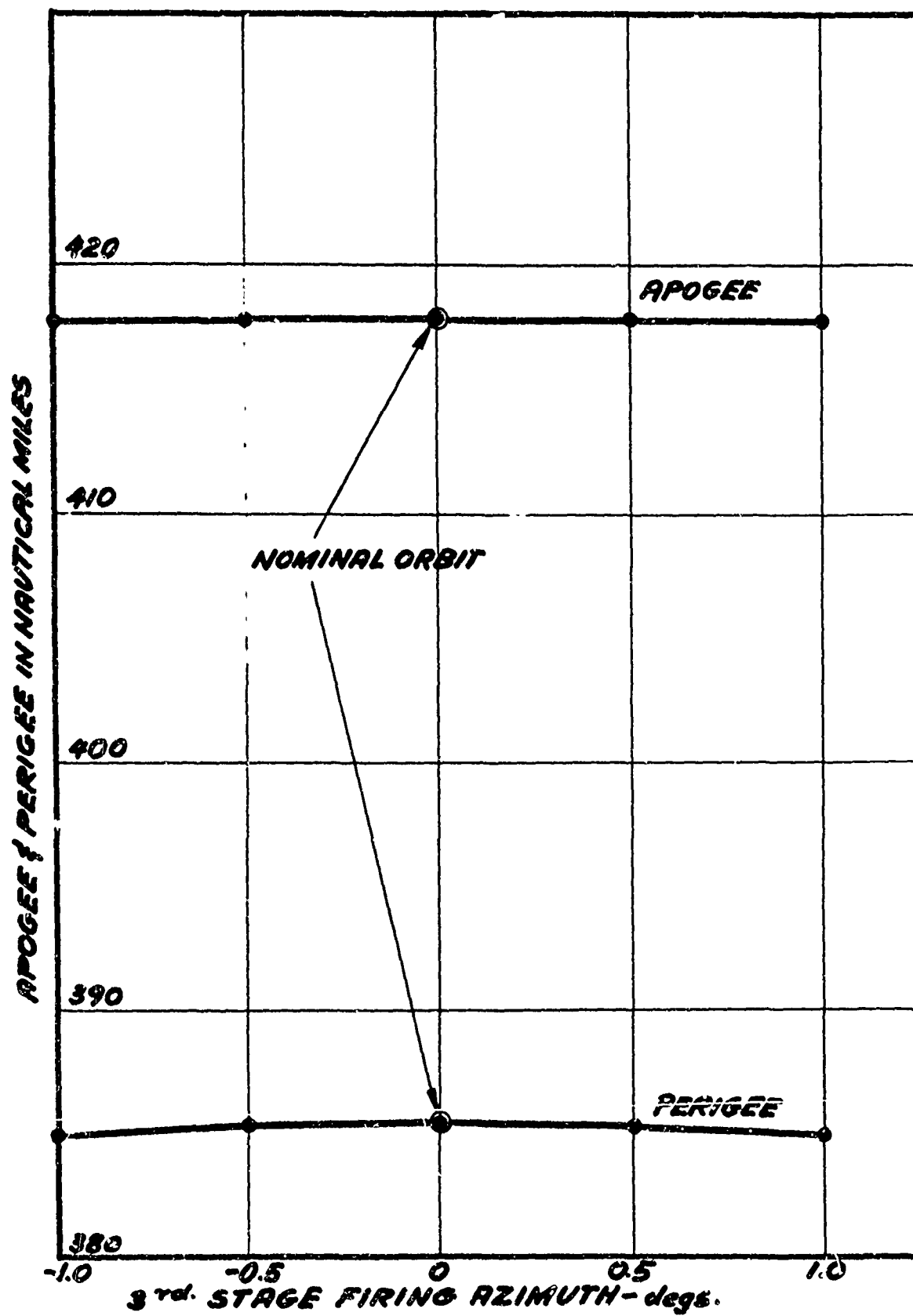
**FIGURE 3.4.3**

TABLE 3.4.3

3rd Stage Firing Azimuth Cases 3127 - 3130

3rd Stage Firing Azimuth	3rd Stage Ignition		3rd Stage B. O.			Orbit Parameters	
	Height	Path Angle	Height	Abs. Vel.	Path Angle	Perigee	Apogee
degrees	N.M.	degrees	N.M.	ft/sec	degrees	N.M.	N.M.
-1.0	415.76	0.26	415.80	24,454.	0.11	385.04	417.73
-0.5	415.76	0.26	415.80	24,454.	0.11	385.40	417.76
0.0	415.76	0.26	415.80	24,454.	0.11	385.52	417.77
0.5	415.76	0.26	415.80	24,454.	0.11	385.41	417.75
1.0	415.76	0.26	415.80	24,454.	0.11	385.04	417.73

3.4.4 3rd Stage - Specific Impulse

The results are shown in graph 3.4.4. The expected dispersion is not known and we looked at the range ± 10 seconds about the nominal value of 290 seconds. As shown positive errors are not significant since the perigee remains constant while the apogee increases. Negative errors will yield a critical below which the perigee will be below 100 N.M. Extrapolation of the curve shows this value to be approximately 273 seconds. Thus negative dispersion of less than 17 seconds will guarantee the success of the mission.

3.4.5 3rd Stage - Burning Rate

The results of dispersion in burning rate on the nominal orbit are shown in graph 3.4.5. Quantitative information about the dispersion was again unknown and we estimated ± 3 lbs/sec about the nominal value of 32 lbs/sec. As shown, the sensitivity to this parameter is very small and will not significantly affect the success of the mission.

3.4.6 Third Stage Weight of Fuel (P 4.6)

This parameter was varied keeping constant the fuel burning rate and the structural weight of the first stage. The results appear in Table 3.4.6 and Figure 3.4.6. As in the previous first and second stage parameters describing the weight of fuel, we obtain an hyperbolic behaviour with two

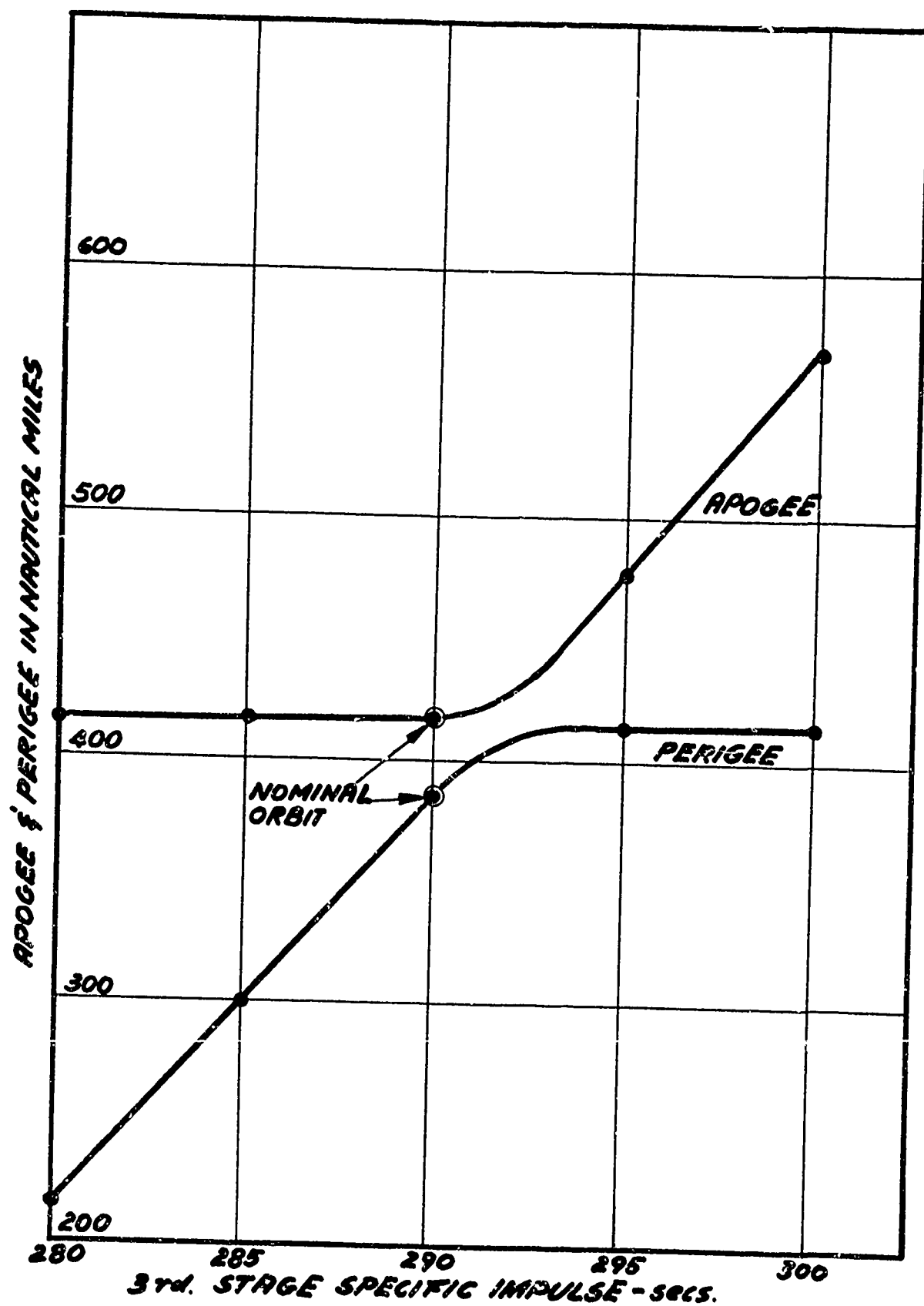
**FIGURE 3.4.4**

TABLE 3.4.4
3rd Stage Specific Impulse Cases 3109 - 3122

3rd Stage Specific Impulse seconds	3rd Stage Ignition		3rd Stage B. O.			Orbit Parameters	
	Height N.M.	Path Angle degrees	Height N.M.	Abs. Vel. ft/sec	Path Angle deg	Perigee N.M.	Apogee M.
300	415.76	0.26	415.80	24,735.	0.12	415.37	568.36
295	415.76	0.26	415.80	24,595.	0.12	414.80	477.42
290	415.76	0.26	415.80	24,454.	0.11	385.52	417.77
285	415.76	0.26	415.80	24,314.	0.11	300.58	416.29
280	415.76	0.26	415.80	24,173.	0.11	216.78	416.07

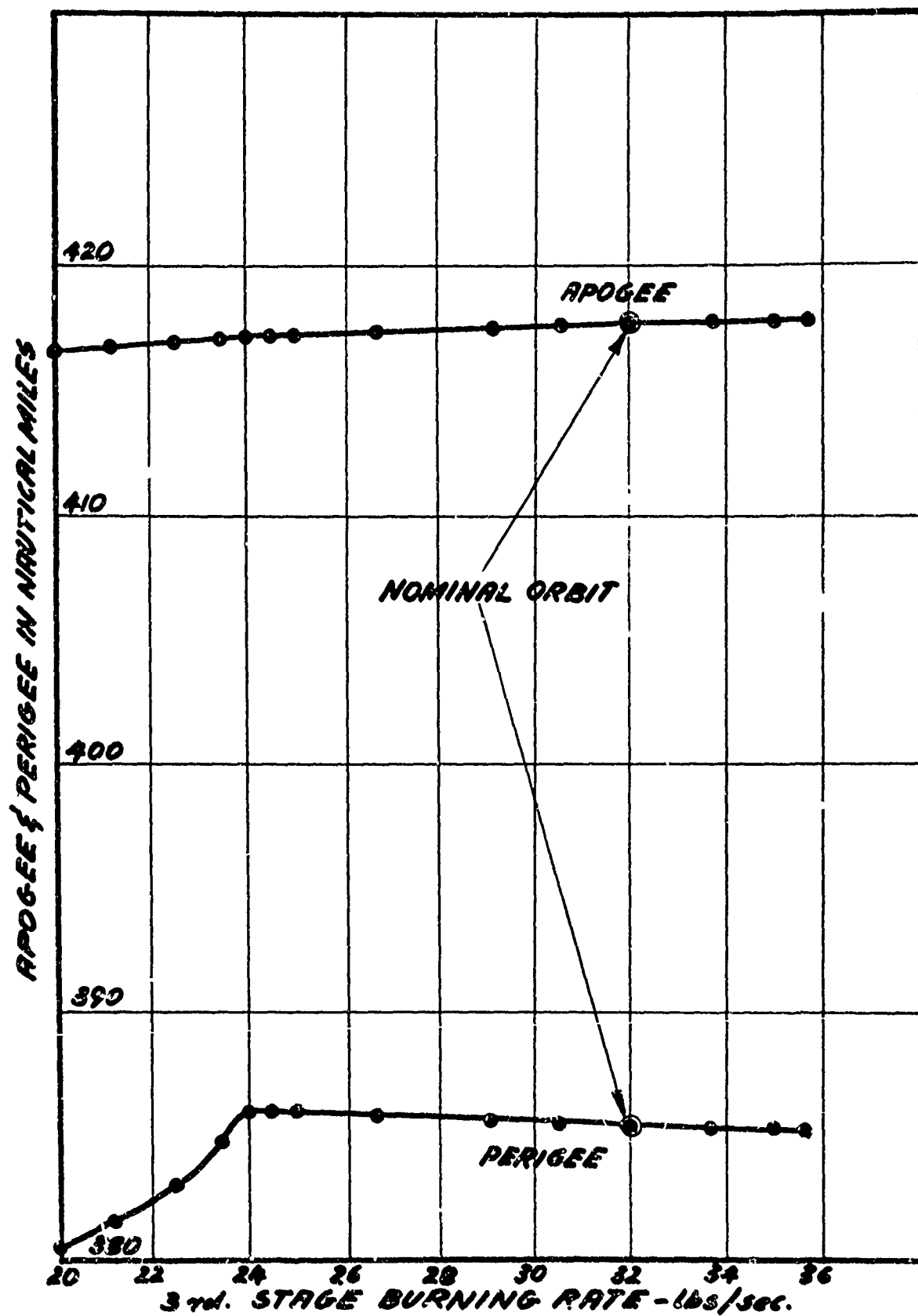


FIGURE 3.4.5.

TABLE 3.4.5

3rd Stage Burning Rate

Cases 3115-3118, 4177-4183

3rd Stage Burning Rate	3rd Stage Ignition		3rd Stage B. O.			Orbit Parameters	
	Height	Path Angle	Height	Abs. Vel.	Path Angle	Perigee	Apogee
lbs/sec	N.M.	degrees	N.M.	ft/sec	degrees	N.M.	N.M.
20.0	415.76	0.26	415.81	24 444	0.08	380.34	416.63
21.3	415.76	0.26	415.81	24 446	0.09	381.56	416.77
22.5	415.76	0.26	415.81	24 449	0.09	383.11	416.94
23.5	415.76	0.26	415.81	24 452	0.09	384.92	417.10
24.0	415.76	0.26	415.81	24 454	0.10	386.06	417.22
24.5	415.76	0.26	415.81	24 454	0.10	386.03	417.25
25.0	415.76	0.26	415.81	24 454	0.10	385.99	417.29
26.7	415.76	0.26	415.81	24 454	0.10	385.85	417.43
29.1	415.76	0.26	415.80	24 454	0.11	385.69	417.60
30.5	415.76	0.26	415.80	24 454	0.11	385.60	417.69
32.0	415.76	0.26	415.80	24 454	0.11	385.52	417.77
33.7	415.76	0.26	415.80	24 454	0.12	385.43	417.86
35.0	415.76	0.26	415.80	24 454	0.12	385.37	417.94
35.6	415.76	0.26	415.80	24 454	0.12	385.34	417.96

asymptotes. If w_3 represents the weight of fuel in the third stage in pounds, the apogee and perigee can be approximated as follows:

$$\begin{array}{lcl} \text{Apogee} & = & \begin{cases} 414.5 + 2.3 (w_2 - 162), & w_2 \leq 162 \text{ lbs} \\ 414.5 + 12.5 (w_2 - 162), & w_2 \geq 162 \text{ lbs} \end{cases} \\ \text{in N.M.} & & \end{array}$$

$$\begin{array}{lcl} \text{Perigee} & = & \begin{cases} 411.5 - 13.0 (w_2 - 162), & w_2 \leq 162 \text{ lbs} \\ 411.5 - 2.07 (w_2 - 163), & w_2 \geq 163 \text{ lbs} \end{cases} \\ \text{in N.M.} & & \end{array}$$

Consequently a variation of one pound in the third stage weight of fuel causes the following variations in the orbit parameters:

$$\Delta \text{ apogee} = \pm 2.3 \text{ N.M. (nominal 417.7 N.M.)}$$

$$\Delta \text{ perigee} = \pm 13.0 \text{ N.M. (nominal 385.5 N.M.)}$$

If the apogee is as expected not very sensitive to the variations in this parameter, the perigee decreases at a rate of 13 N.M. per pound when the amount of fuel is reduced. Note also that a weight of fuel of 162 pounds would produce an almost perfectly circular orbit. It would, of course, be preferable to employ a liquid or hybrid rocket motor for the third stage so that eccentricity could be controlled by burning time. However where it is not possible to shut off the motor this curve and that of Appendix "B" showing the effect of dispersion in payload weight suggest the use of jettisonable ballast as a means of flight control.

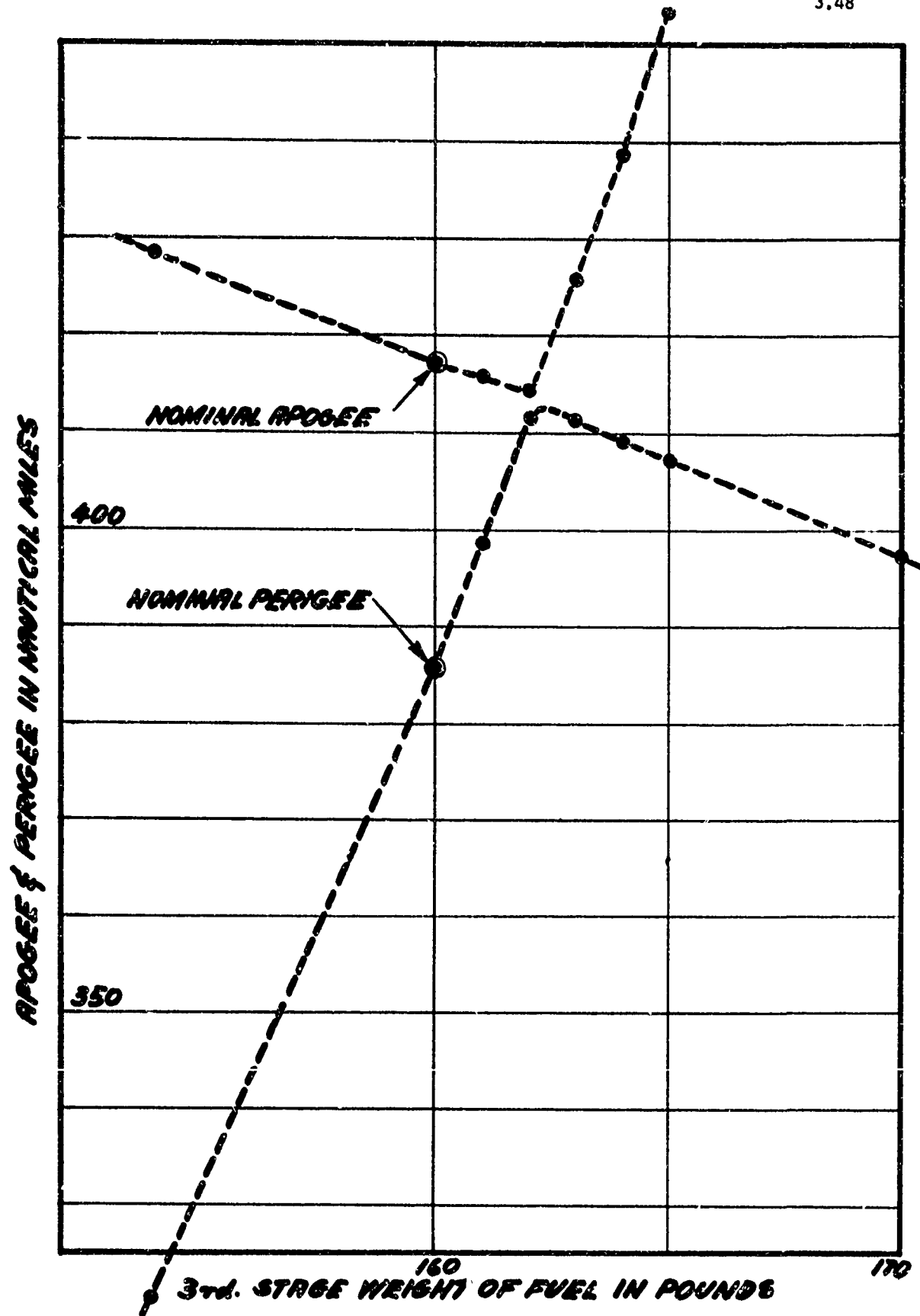


FIGURE 3.4.6

TABLE 3.4.6

Third stage Weight of Fuel (P 4.6) Cases 3145,
3203, 3210, 3217, 3224, 3110 - 3114

3rd Stage Weight of Fuel	Orbit Parameters	
	Perigee	Apogee
pounds	N.M.	N.M.
150	257.35	439.54
155	320.25	428.48
160	385.52	417.17
161	398.87	415.70
162	411.56	414.38
163	411.36	426.07
164	409.29	438.69
165	407.29	453.50
170	397.23	524.14

4. DISCUSSION AND OPTIMIZATION

4.1 Discussion of the Results

The results of chapter 3 will be discussed according to the following scheme:

1. Gun (P 1.1, P 1.2)
2. First stage Ignition (P 2.1)
3. First stage Rocket Motor (P 2.4, P 2.5, P 2.6)
4. Second stage Ignition (P 3.1, P 3.2, P 3.3)
5. Second stage Rocket Motor (P 3.4, P 3.5, P 3.6)
6. Third stage Ignition (P 4.1, P 4.2, P 4.3)
7. Third stage Rocket Motor (P 4.4, P 4.5, P 4.6)

1) Gun

The gun parameters are the gun muzzle velocity and elevation. The study of the gun parameters in Figures 3.1.1 and 3.1.2 has shown that the muzzle velocity need not be critically adjusted, but must only be kept greater than a certain minimum velocity which is linearly related to the desired minimum perigee.

The gun elevation is a more critical parameter; its adjustment mainly depends on how circular the orbit is desired and how high the minimum perigee is required to be. The orbit parameters can still be improved with the following values:

gun muzzle velocity	:	6000 feet/sec
gun elevation	:	33.62 degree

2) First stage Ignition

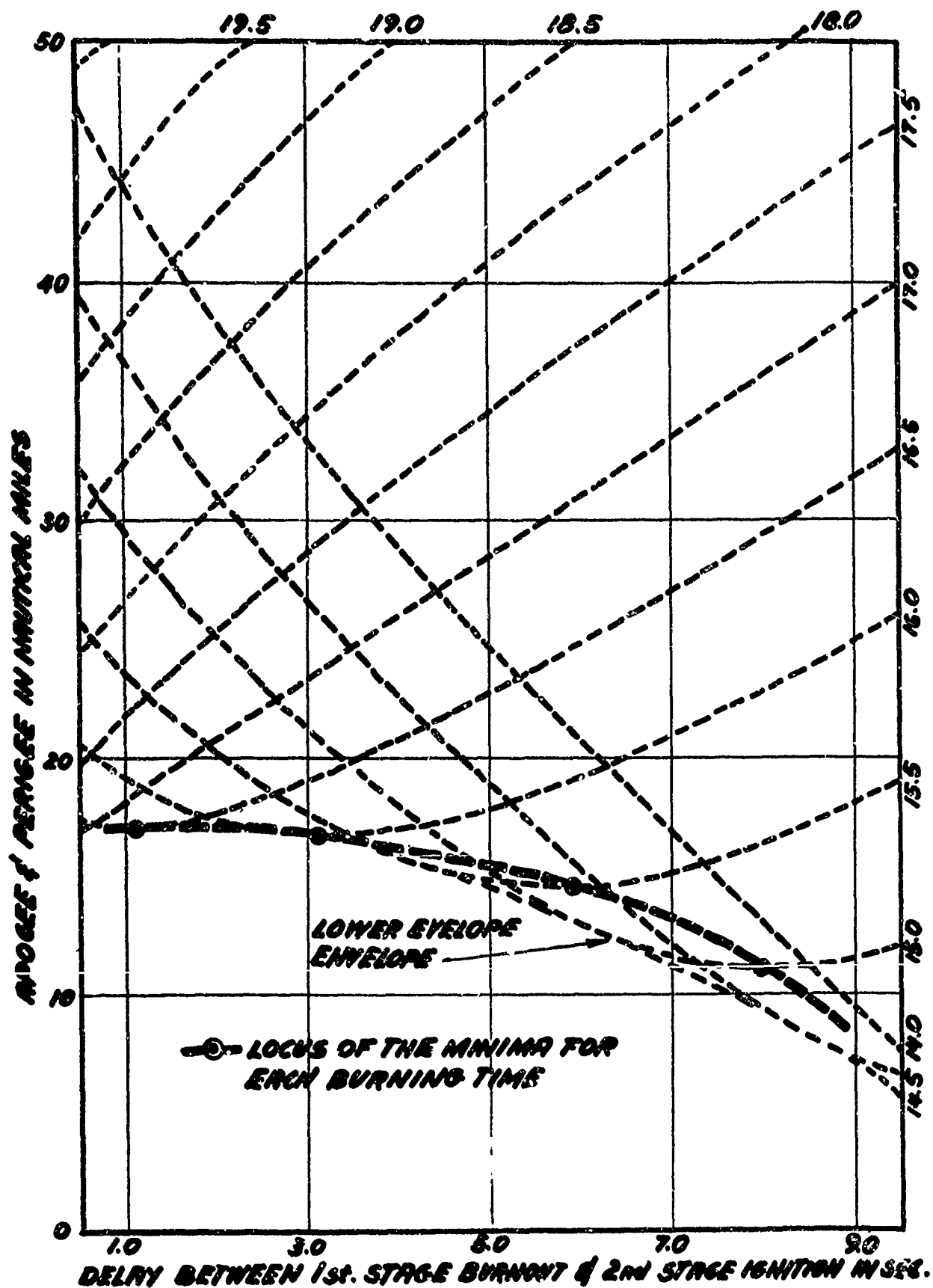
The only first stage ignition parameter is the ignition time. Here it was found that the first stage ignition should be delayed from 14.61 to 16.0 second in order to obtain a higher perigee and a more circular orbit.

However since there is only half a second between first stage burnout and second stage ignition, to delay the first stage ignition is equivalent to also delay second stage ignition. This fact would call for a parametric study where the first stage ignition time, the delay between first stage burnout and second stage ignition and the second stage ignition time are simultaneously varied.

3) First stage Rocket Motor

The first stage rocket motor parameters considered in this section are the propellant specific impulse, burning rate and weight. All parameters optimize the orbit but the burning rate. From Figure 3.2.3 a burning rate of 96 pounds per second instead of 107.9 pounds per second would be recommended; this is equivalent to burning the 1619 pounds of fuel in 16.86 seconds instead of 15 seconds.

However we know that if the burning rate is decreased the second stage ignition will have to be delayed. Therefore, a parametric study was undertaken with the first stage burning time and the delay between first stage burnout and second stage ignition as parameters. In Figure 4.1 was plotted the difference between apogee and perigee as a function of the delay between first stage burnout and second stage ignition for several values of the propellant burning time (first stage). The minimum of each curve (for a fixed burning time) can be easily determined; the locus of all these points is shown on the graph. It can easily be seen that a more circular orbit is obtained as the second stage ignition time is delayed and the first stage burning rate

**FIGURE 4.1**

increased. The authors did not look for a minimum but, if necessary, some more computer runs could yield the information; it was deemed that a burning time of 14 second (that is a burning rate of 115.64 lb/sec) was a lower limit on the burning time of the 1619 pounds of fuel. Table 4.1 summarizes the results and the appropriate conclusions can be drawn by the reader.

Finally if the specific impulse and the weight of the propellant (Figure 3.2.2 and 3.2.4) are kept greater than a certain value function of the perigee, a minimum perigee will be guaranteed; any increase above the minimum specific impulse or fuel weight will only contribute to accentuate the eccentricity of the orbit.

4) Second stage Ignition

The second stage ignition parameters are the ignition time of the second stage rocket motor and the firing pitch and yaw angles of the vehicle when the ignition occurs.

The ignition time of the second stage was arbitrarily made to occur 0.5 second after first stage burn-out. The effect of increasing this delay was studied and Fig. 3.3.1 shows that the orbit can be improved if the delay is increased to 8.5 seconds. The delay would need to be adjusted further to allow for variation in burning rate of the first stage fuel as discussed above.

As far as the firing attitude of the vehicle's axis is concerned, the orbit parameters are much more sensitive to the pitch than to the yaw angle. As an instance of this fact a variation of 2 degrees in

TABLE 4.1

1st stage Fuel		Delay between 1st stg burnout and 2nd stg ignition	Orbit Parameters	
Burning Time	Burning Rate		Perigee	Apogee- Perigee
sec	lbs/sec	second	N.M.	N.M.
20.	80.95	0.5	405.99	48.87
19.5	83.02	0.5	406.83	41.74
19.0	85.21	0.5	407.53	35.67
18.5	87.51	0.5	408.00	29.80
18.0	89.94	0.5	408.13	24.37
17.5	92.51	0.5	407.67	19.84
17.	95.23	0.5	406.18	17.10
16.5	98.12	1.2	404.90	17.20
16.	101.18	3.1	405.53	16.70
15.5	104.45	5.85	406.56	14.60
15.	107.93	8.1		11.1
14.5	111.63	9.5 (not opt)	406.17	6.62
14.	115.64	9.5 (not opt)	402.61	7.61

yaw will bring down the perigee by approximately 6 nautical miles while the same variation in pitch brings it down by 75 nautical miles. Nevertheless if only a minimum perigee of 100 nautical miles is required, the mission can easily suffer a 7 to 8 degree error in the firing pitch angle provided no significant extra error arises from the other trajectory parameters (unlikely). As a consequence and as was already pointed out, the fact that this parameter is not controlled makes it the weakest link of the mission.

5) Second stage Rocket Motor

The second stage rocket motor parameters are the specific impulse, burning rate and weight of the propellant. If the specific impulse and the weight of the propellant (Figures 3.3.4 and 3.3.6) are kept greater than a certain value which is a function of the perigee, a minimum perigee will be guaranteed; any further increase in the specific impulse or fuel weight will only contribute to accentuate the eccentricity of the orbit.

More interesting is the burning rate of the propellant; nominally fixed at 40 pounds per second, Figure 3.3.5 clearly shows that the orbit parameters can be quite improved if a burning rate of 11.4 pounds per second is chosen, that is a burning time of 35.1 second. However the value of the perigee drops on both sides of the optimal 11.4 pounds per second; as a consequence it would be advisable to operate with a burning rate greater than 11.4 pounds per second since between 12 and 44 pounds per second, the perigee varies linearly with the burning rate (-3 nautical miles for an increase of 4 pounds per second in the burning rate).

6) Third stage Ignition

The third stage ignition parameters are the time at which the third stage rocket motor is ignited and the firing pitch and yaw angles of the vehicle at ignition.

All parameters yield a maximum perigee for the following values:

Ignition	:	608 seconds from launch
Firing Yaw	:	+0.0 degree
Firing Pitch	:	-0.3 degree

The firing yaw seems to have almost no effect on the orbit parameters; this is certainly true for quite large an error (may be up to 10 degrees) and it would be easy to find the maximum permissible error in this parameter. As expected the firing pitch and the ignition time are critical and the range of error can be determined by the minimum desired perigee.

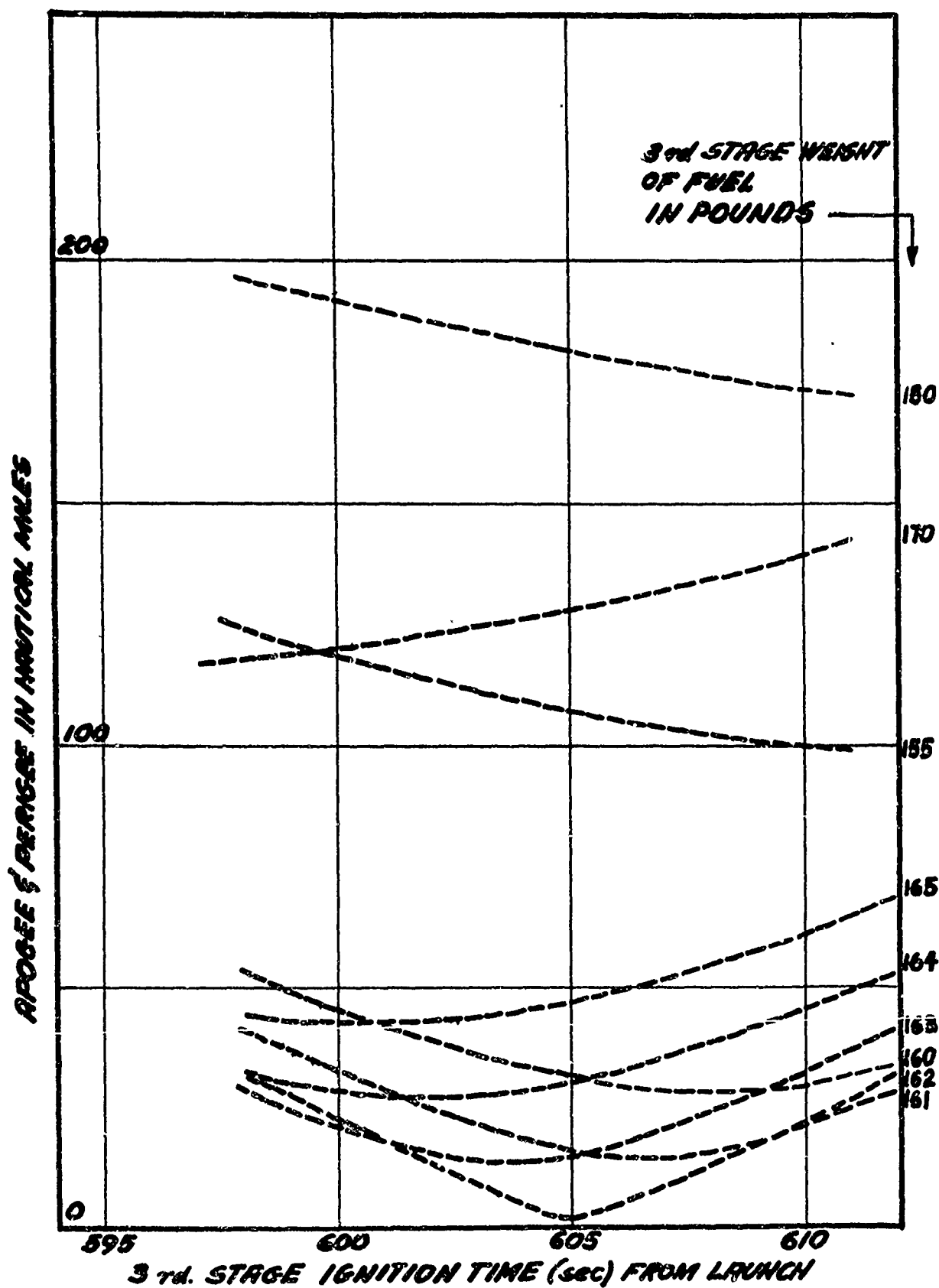
7) Third stage Rocket Motor

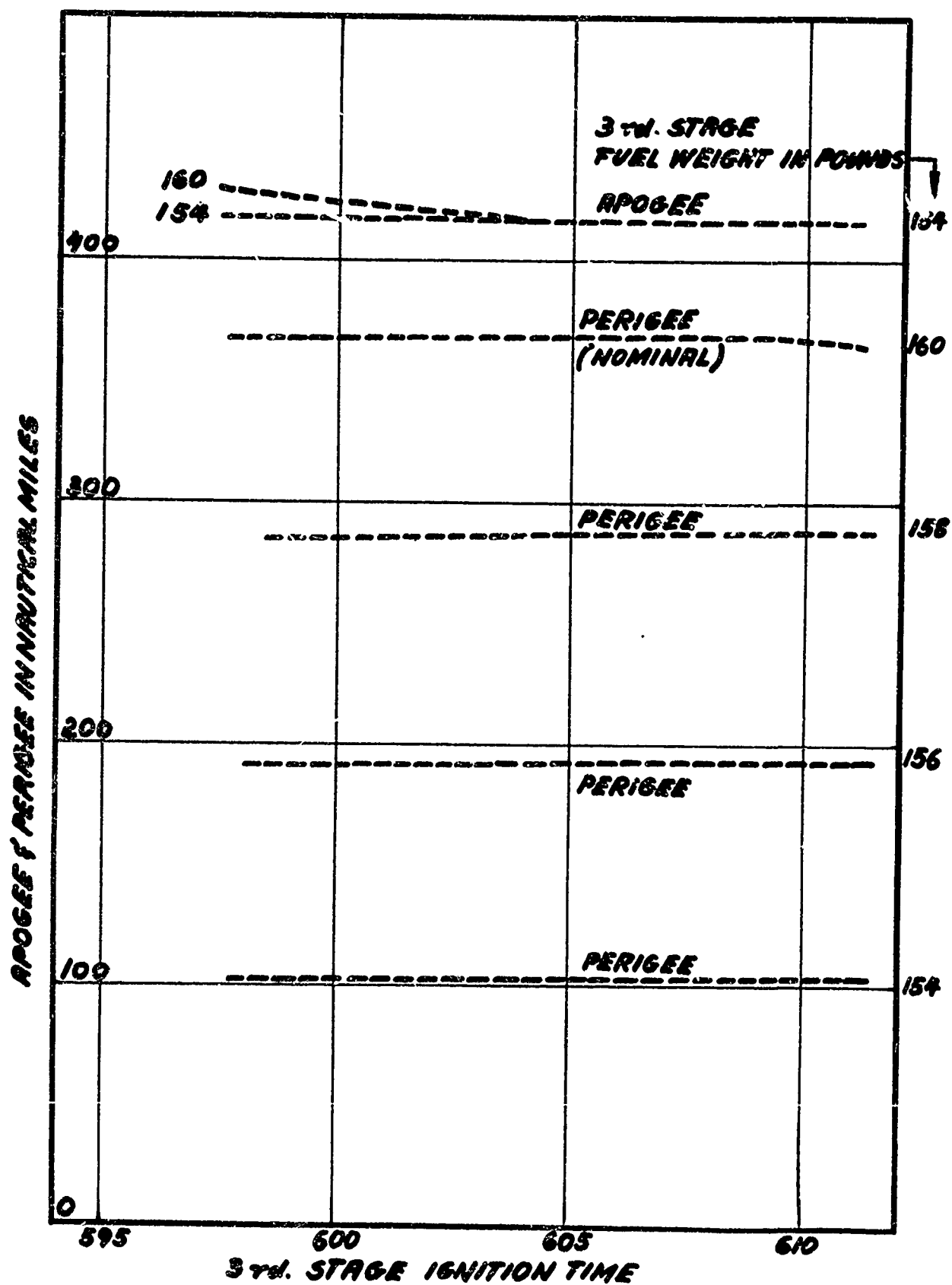
The third stage rocket motor parameters are the specific impulse, burning rate and weight of the propellant. Again if the specific impulse of the propellant is kept greater than a certain value, a minimum perigee will be guaranteed; too high a specific impulse will not alter the perigee but accentuate the eccentricity of the orbit.

The variations of the orbit parameters with the burning rate present a special feature: it seems that a discontinuity takes place in the slope of the curve representing the perigee as a function of the burning rate at 24 pounds per second. Below 24 pounds per sec., the perigee

decreases very rapidly (6 nautical miles for 2 pounds per second); however for burning rates greater than 24 pounds per second, the perigee decreases at a rate of 0.07 nautical mile for an increase of one pound per second in the burning rate. Similarly the apogee is not very sensitive to the burning rate in the range over which the orbit parameters are plotted (20 to 36 pounds per second). As a practical conclusion, it can be said that the orbit parameters are not significantly affected when the burning rate lies between 24 and 36 pounds per second.

On the other hand the weight of fuel is very significant as can easily be seen in Figure 3.4.6. A maximum perigee is obtained with 162 pounds of fuel, that is 2 pounds above the nominal weight of fuel. This brings up the natural question: can some control be devised to effect some changes in the orbit parameters by varying the amount of fuel burnt. In order to get some information in that sense two parametric studies were undertaken. The first one (Figure 4.2) shows the difference between the apogee and the perigee as a function of the ignition time and the weight of fuel; in that case the parameter which can be controlled is the ignition time. The second parametric study (Figure 4.3) again shows the difference between the apogee and the perigee as a function of the ignition time and the weight of fuel burnt, (that is the third stage of the vehicle carries 160 pounds of fuel) but there is control not only over the ignition time, but also over the burnout; this amounts to burn all or only part of the propellant contained in the third stage of the vehicle.

**FIGURE 4.2**

**FIGURE 4.3**

The results of the first study shows that a nominal 162 pounds of fuel would be recommended with the ignition occurring 605 seconds from launch. Nevertheless this ignition time can oscillate by 5 seconds in both directions about 605 seconds without significantly affecting the eccentricity (almost 1) and perigee of the orbit.

The second study shows that the apogee (as expected) is not affected, but the amount of fuel (160 pounds) was not sufficient to effect the type of control described earlier; it also shows that between 598 and 611 seconds the ignition time does not significantly affect the orbit parameters. No further studies were attempted but serious thoughts should be given to this technique of control especially if the third stage propellant is liquid.

Nevertheless all the above mentioned studies should have been undertaken after a careful "optimization" of the orbit parameters with respect to the trajectory parameters; a numerical technique to accomplish this optimization is described in the next section.

4.2 ORBIT OPTIMIZATION

In this section we will discuss and describe a numerical technique which will optimize the type of orbit achievable through a Martlet IV trajectory. By optimize we mean an orbit which will have the highest possible perigee and will be as circular as possible. A further constraint on the perigee, P , will be

$$P \geq 100 \text{ N.M.}$$

A cost criterion which would represent such a requirement could be

$$J = \frac{A - P}{P}, \quad P \geq 100$$

where A is the apogee or

$$J = \log(A - P) - \log(P - 100)$$

and the optimum orbit should be such as to minimize the chosen cost criterion.

To accomplish this, the parameters defining A and P will be listed below together with their range of definition or of possible variation. These ranges will form the parameter constraint set, Ω .

Gun Parameters

x_1 - Gun angle	(degrees)	[1, 89]
x_2 - Muzzle velocity	(ft/sec)	[2000, 6000]

1st Stage parameters

x_3 - Ignition time	(secs)	[0, 00]
x_4 - Specific impulse	(secs)	[0, 280]

x_5 - Burning rate	(lbs/sec)	[1,120]
x_6 - Weight of fuel	(lbs)	(0,1650]

2nd Stage parameters

x_7 - Ignition time	(secs)	(0,00)
x_{10} - Specific impulse	(secs)	[0,290]
x_{11} - Burning rate	(lbs/sec)	[1,50]
x_{12} - Fuel weight	(lbs)	(0,450]

3rd Stage parameters

x_{13} - Ignition time	(secs)	[0,00)
x_{14} - Firing pitch	(degrees)	[-45,+45]
x_{15} - Firing yaw	(degrees)	[-10,+10]
x_{16} - Specific impulse	(secs)	(0,270]
x_{17} - Burning rate	(lbs/sec)	[1,35]
x_{18} - Fuel weight	(lbs)	(0,165]

Parameters x_8 and x_9 which are the 2nd stage ignition pitch and yaw respectively are not included in the optimization as there is no control over them. They will therefore be assumed to orient the vehicle parallel to the absolute velocity vector. Further restrictions must be imposed on the parameters x_7 and x_{13} , the ignition times,

$$(1) \quad x_7 \geq 0.5 + x_3 + \frac{x_6}{x_5}$$

$$(2) \quad x_{13} \geq 0.5 + x_7 + \frac{x_{12}}{x_{11}} + 200$$

The first inequality which must be satisfied before the second is simply a statement that the second stage ignition time, x_7 , be greater than the first stage ignition time, x_3 , plus the burning time, $\frac{x_6}{x_5}$, plus a minimum arbitrary time, 0.5 seconds, after first stage burnout. The second inequality can be similarly explained except that 200 seconds must be allowed for the guidance system to correct the pitch and yaw before third stage ignition.

The numerical technique suggested to optimize the orbit in the sense discussed earlier is a hill-climbing technique. The optimization problem can be written as follows:

Find the parameters x_1 to x_7 and x_{10} to x_{18} from the constraint set, Ω , which satisfy

$$x_7 \geq 0.5 + x_3 + \frac{x_6}{x_5}$$

$$x_{13} \geq 0.5 + x_7 + \frac{x_{12}}{x_{11}} + 200$$

and which absolutely minimize

$$J = \log(A - P) - \log(P - 100)$$

Clearly J is a function of all x_i thus

$$J = J(\underline{x})$$

where $\underline{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_{18} \end{bmatrix}$

Suppose we start with the value \underline{x}_0 or $J(\underline{x}_0)$ where $\underline{x}_0 \in \Omega$ and satisfies the inequality constraints, and vary each x_i about the value x_{i0} by $\pm \Delta x_i$, a small quantity, keeping the rest of the x_i fixed. If the corresponding change in J is negative for either $+\Delta x_i$ or $-\Delta x_i$, then change x_i by $+\Delta x_i$ or $-\Delta x_i$ whichever yields a larger decrease in J . If there is no change in J or if it is positive then x_i is left unchanged. This is repeated for each x_i until J remains unchanged.

The convergence of this procedure is guaranteed if the Δx_i are sufficiently small. Their value can be obtained from the sensitivity analysis of section 3.0. The computer programming required to put this technique into effect is relatively simple as it requires a main program performing the search described above plus an apogee - perigee subroutine which is simply McKee's program for orbit analysis. The computing time may be quite large depending on the number of iterations required for the convergence of J , nevertheless this technique would give a much better description of the orbital capabilities of the Martlet IV than the one used in this report.

5.0 CONCLUSIONS

The aim of this report was to study the variations in a nominal orbit resulting from errors in the trajectory parameters. Although caution must be used in extending the conclusions drawn to cover other trajectories, the nominal orbit used was a typical one of the low-gun angle type and the curves of Appendix C give a good indication of the effect of dispersion in the delivered performance.

The parameters to which the orbit is most sensitive are muzzle velocity, propellant specific impulse, fuel weight, elevation angle during thrusting, and payload weight. The assumed probable errors in each of these parameters are not likely to prevent orbit, i.e. yield a perigee less than 100 N.M., but combined errors could result in too low an orbit.

Careful design of the rocket motors and reliable test data on their performance and on the characteristics of the fuel are obviously desirable in planning the details of a launch. If real-time data can be made available on attained muzzle velocity, flight path angle and rocket motor thrust it seems that ample time is available for the flight controller to evaluate them and execute compensation commands. The preferred controls are 3rd stage ignition attitude, ignition time, burning time (in the case of a liquid or hybrid third stage) and ejection of ballast.

An attitude control system which could combat tip-off disturbance at first stage separation would of course be desirable. It had been

assumed that the increased cost and complexity of such a control system could be avoided by careful attention to the design of the stage separation mechanism. Although the present analysis shows that a longer delay is advisable there still may not be enough time available for a simple system to be effective. Further study of this is recommended.

It is also recommended that an optimization exercise of the type discussed in section 4.2 be carried out and its usefulness evaluated. It is evident that a thorough pre-flight analysis of the effects of variation in performance would not only be helpful in the planning of a mission but would also be useful during the design of the vehicle. It is also believed that an optimization program of that type could provide a set of curves which would be adequate for real-time flight control purposes. It would be advisable to include in the optimization program sub-routines to give directly the dispersion in orbit parameters and plot these with respect to the optimal trajectory.

REFERENCES

1. Bull, G.V., Lyster, D. and Parkinson, G.V., "Orbital and High Altitude Probing Potential of Gun Launched Rockets", Space Research Institute of McGill University, R-SRI-H-R-13, October 1966.
2. Bull, G.V., Aikenhead, B.A. and Palacio, L., "A Gun Launch Target Placement System", Space Research Institute of McGill University, SRI-2-TN-4, August 1966.
3. McKee, R.M., "A Parametric Study of Multi-Stage Gun Launched Rockets", Space Research Institute of McGill University, SRI-H-R-2 and SRI-H-R-5, March 1965.

APPENDIX A

Nominal Trajectory

NOMINAL ORBITAL TRAJECTORY
CASE 3043, BASED ON CASE 1046

(ALL-SOLID MARTLET & LOW GUN ANGLE)

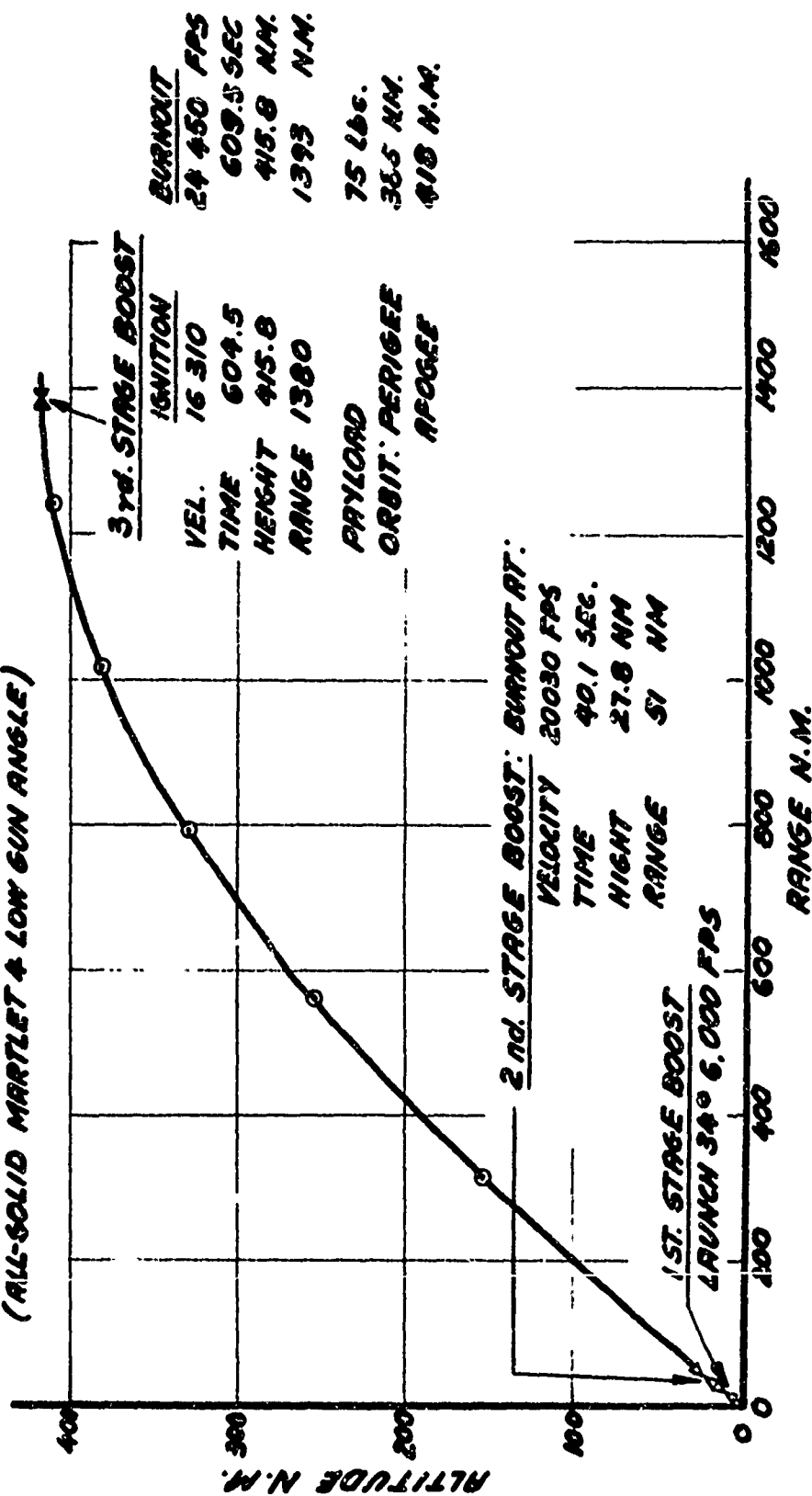


FIGURE 1

HARP TRAJECTORY PROGRAM

PROGRAM - R. M. MCKEE - MK 4 04 JUL 67 CASE 3043 PAGE 1

CASE NUMBER 3043 RUZZLE VELOCITY 6000.
 NO. OF STAGES 3 ELEVATION 34.00
 PAYLOAD WEIGHT 75.00 AZIMUTH 118.23
 AREA 1.478 LONGITUDE -59.48
 STEP SIZE 5. LATITUDE 13.07
 OUTPUT INTERVAL 100. HEIGHT 150.
 PLOT INDICATOR -0

STAGE NUMBER 1 2 3
 IGNITION INDICATOR 2 2 2
 FIRING INDICATOR 1 2 2
 WEIGHT (TOTAL) 2735.0 775.0 275.0
 AREA 1.478 1.478 1.478
 EXHAUST AREA 0.785 0.785 0.785
 BURNING TIME 15.00 10.00 5.00
 IGNITION VALUE 14.61 0.50 564.40
 FIRING ELEVATION 0.00 27.08 0.00
 FIRING AZIMUTH 0.00 3.33 0.00
 MASS FRACTION 0.83 0.80 0.80
 SPECIFIC IMPULSE 280. 290. 290.

TIME	HEIGHT	RANGE	ABSOLUTE VELOCITY	AIRSPEED	PATH ANGLE	ELEVATION	VEHICLE ATTITUDE		RELATIVE VELOCITY VECTOR		PAT-4 AZ
							AZIM	MIND EL	MIND AZ	DYN PRES	
0.00	0.02	0.00	7163.	6000.	27.93						
STAGE 1 IGNITION	- LONGITUDE	-59.319	LATITUDE 12.985								
14.61	6.78	10.77	6048.	4830.	23.51	29.97	7.35	29.97	6.47	42597.17	118.23
29.61	16.05	28.07	13547.	12341.	24.54	27.13	3.33	27.13	7.35	6472.58	118.33
STAGE 1 BURNOUT	- LONGITUDE	-59.058	LATITUDE 12.848								
STAGE 2 IGNITION	- LONGITUDE	-59.045	LATITUDE 12.841								
30.11	16.51	28.97	13529.	12322.	24.50	27.06	3.33	27.08	3.33	2481.58	119.44
40.11	27.75	51.08	20031.	18827.	24.98	27.48	2.26	26.70	2.27	296.85	119.57
STAGE 2 BURNOUT	- LONGITUDE	-58.713	LATITUDE 12.665								
140.11	153.76	314.65	18803.	17522.	21.45						
240.11	254.33	560.78	17847.	16498.	17.46						
340.11	330.22	794.41	17127.	15720.	13.08						
440.11	382.03	1019.56	16634.	15185.	8.38						
540.11	410.12	1239.61	16367.	14893.	3.47						
STAGE 3 IGNITION	- LONGITUDE	-39.553	LATITUDE 1.332								
604.51	415.76	1380.05	16313.	14835.	0.26	-0.00	0.00	0.28	3.21	0.00	122.17
609.51	415.80	1393.49	24454.	22968.	0.11	0.24	0.00	0.12	2.77	0.00	121.04
STAGE 3 BURNOUT	- LONGITUDE	-39.362	LATITUDE 1.215								
	PERIGEE	385.52	APOGEE 417.77								

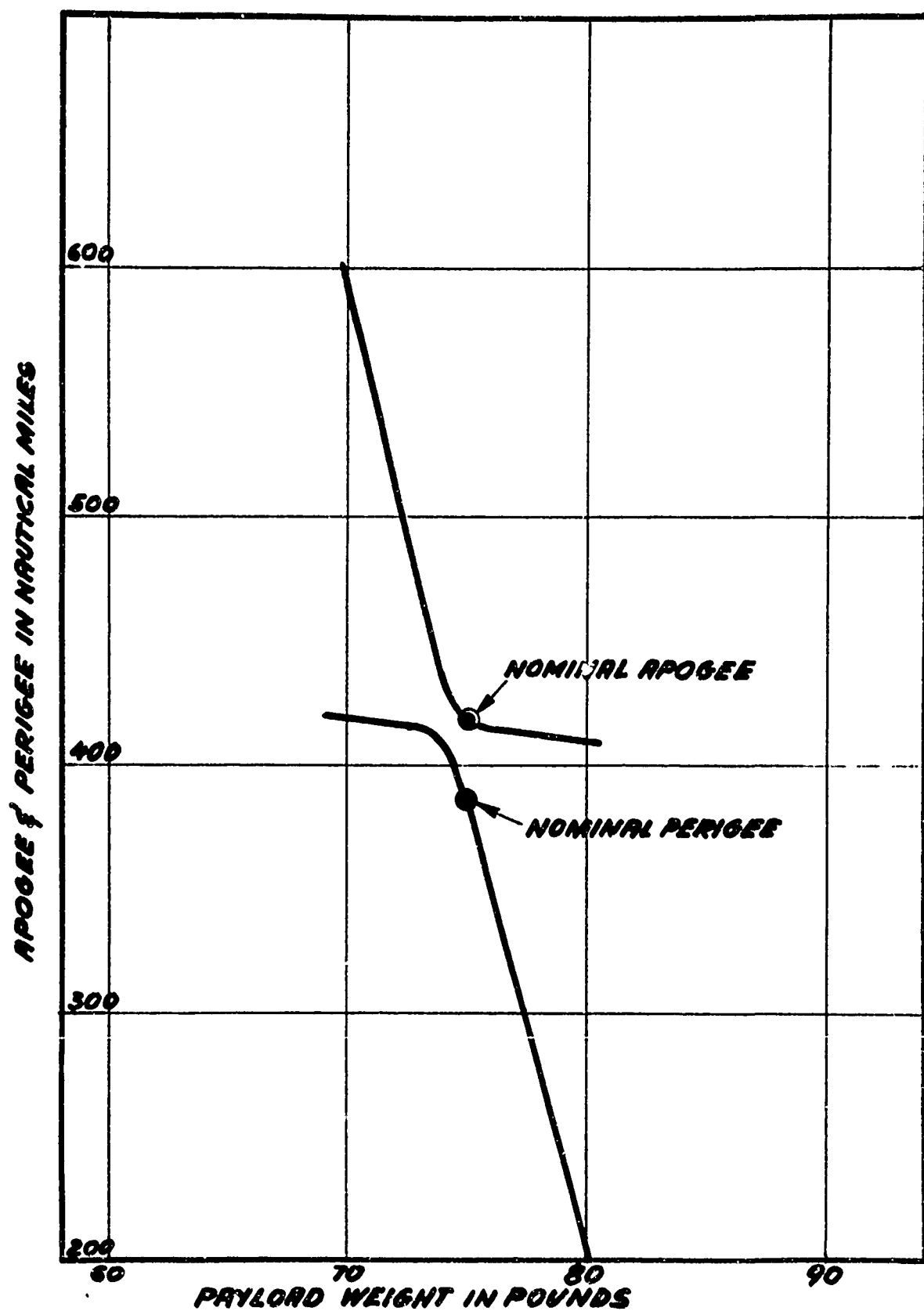
ACCURACY CHECK - U -0.00000250 W -0.00000259 U.W 0.00000003
 UNDERFLOW
 COMPUTING TIME 3.75 SECONDS

APPENDIX B

Orbit Parameters Sensitivity to Payload
Weight Variations

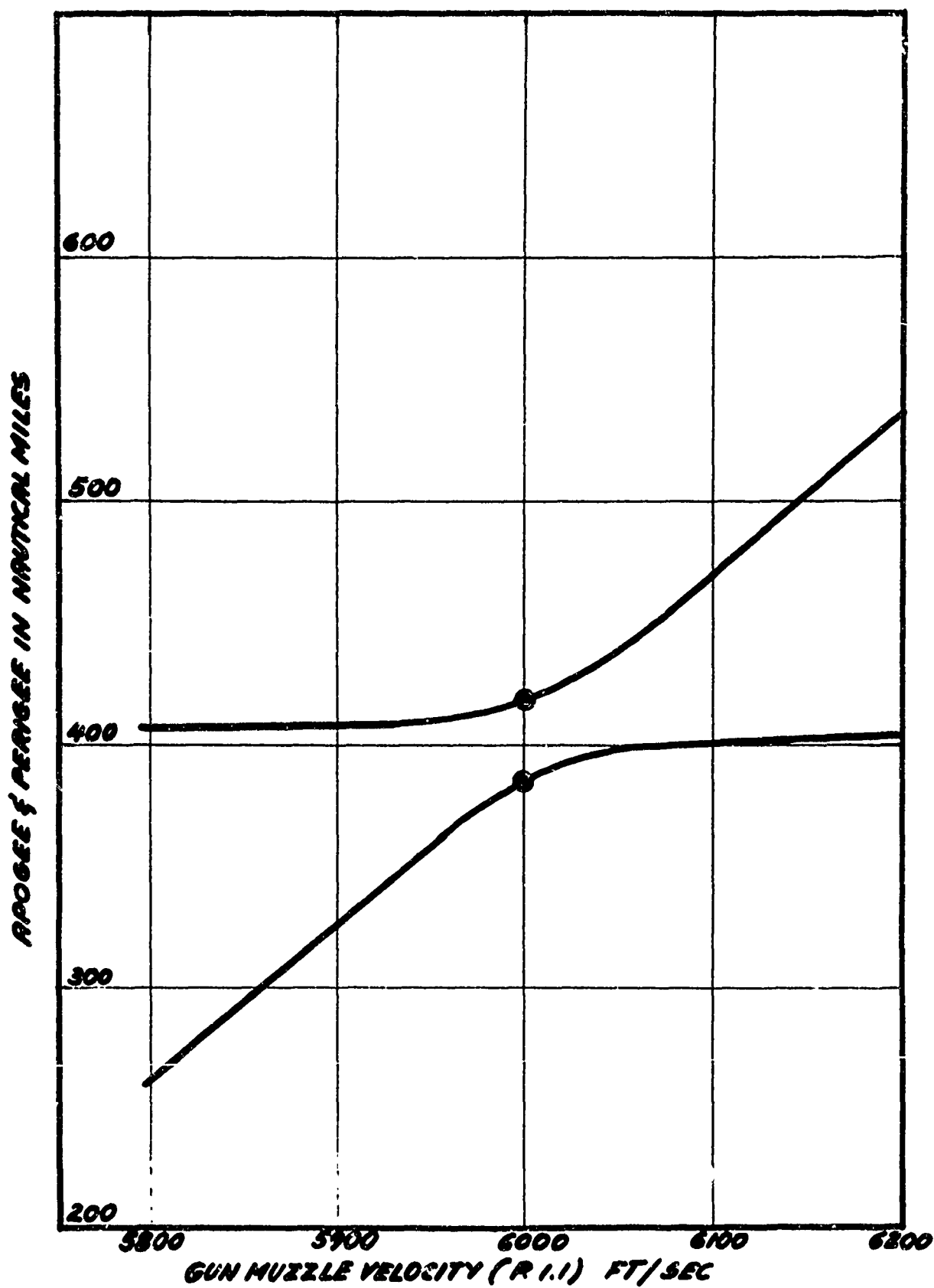
Payload Weight	Orbit Parameter	
	Perigee	Apogee
lbs	N.M.	N.M.
70	419.07	595.63
71	417.78	552.61
72	416.36	510.78
73	414.47	470.43
74	409.77	433.86
75	385.52	417.77
76	349.12	414.74
77	312.20	413.09
78	275.81	411.77
79	240.10	410.56
80	205.12	409.42

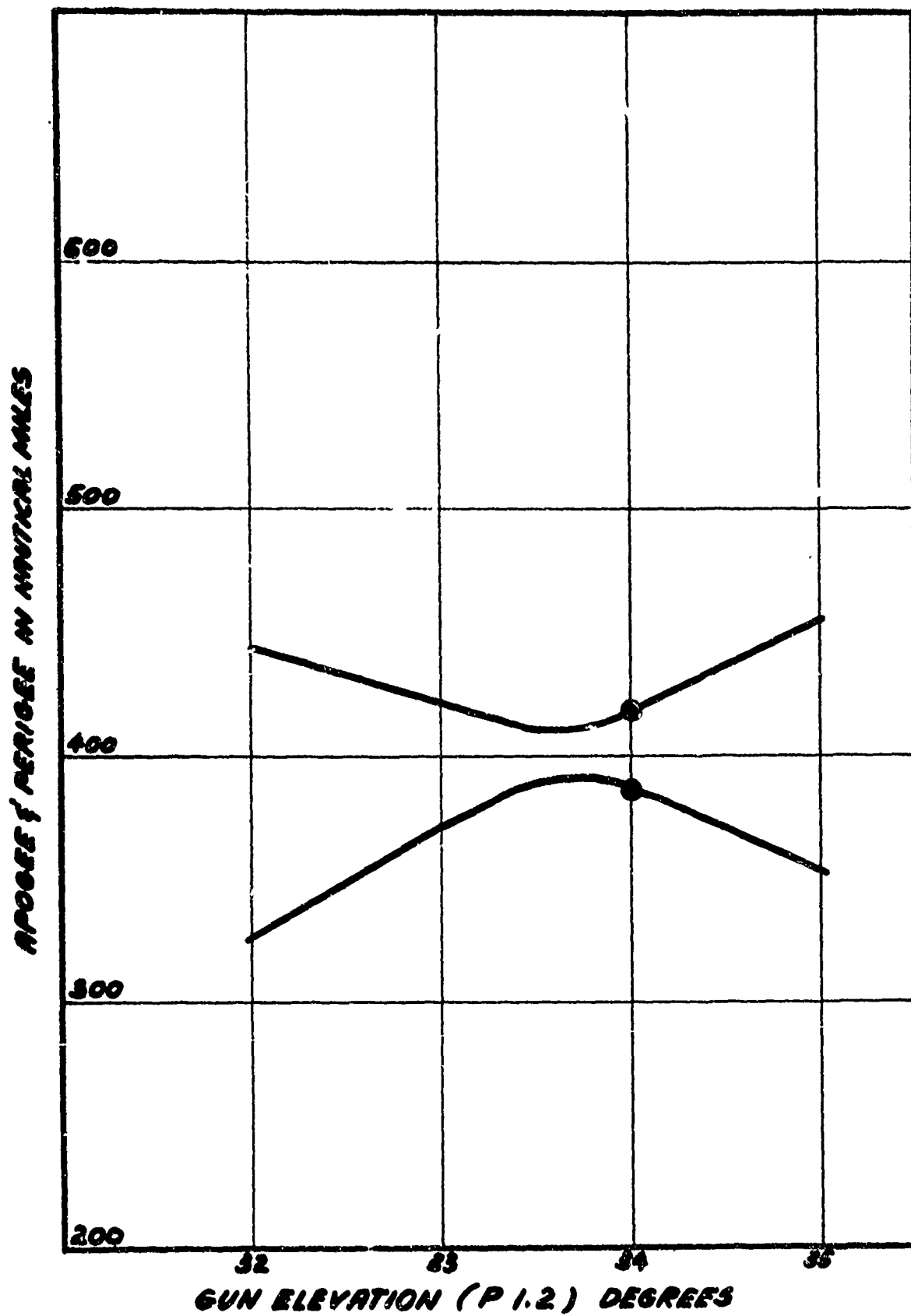
The results appear in Table B and Figure B. Figure shows an hyperbolic behaviour. Below 74 pounds, the perigee remains relatively constant while the apogee increases at a rate of 40.4 nautical miles per pound of payload taken off; above 75 pounds, the apogee remains constant but the perigee decreases at the rate of 40.5 nautical miles for each added pound of payload. Consequently this parameter is very significant.

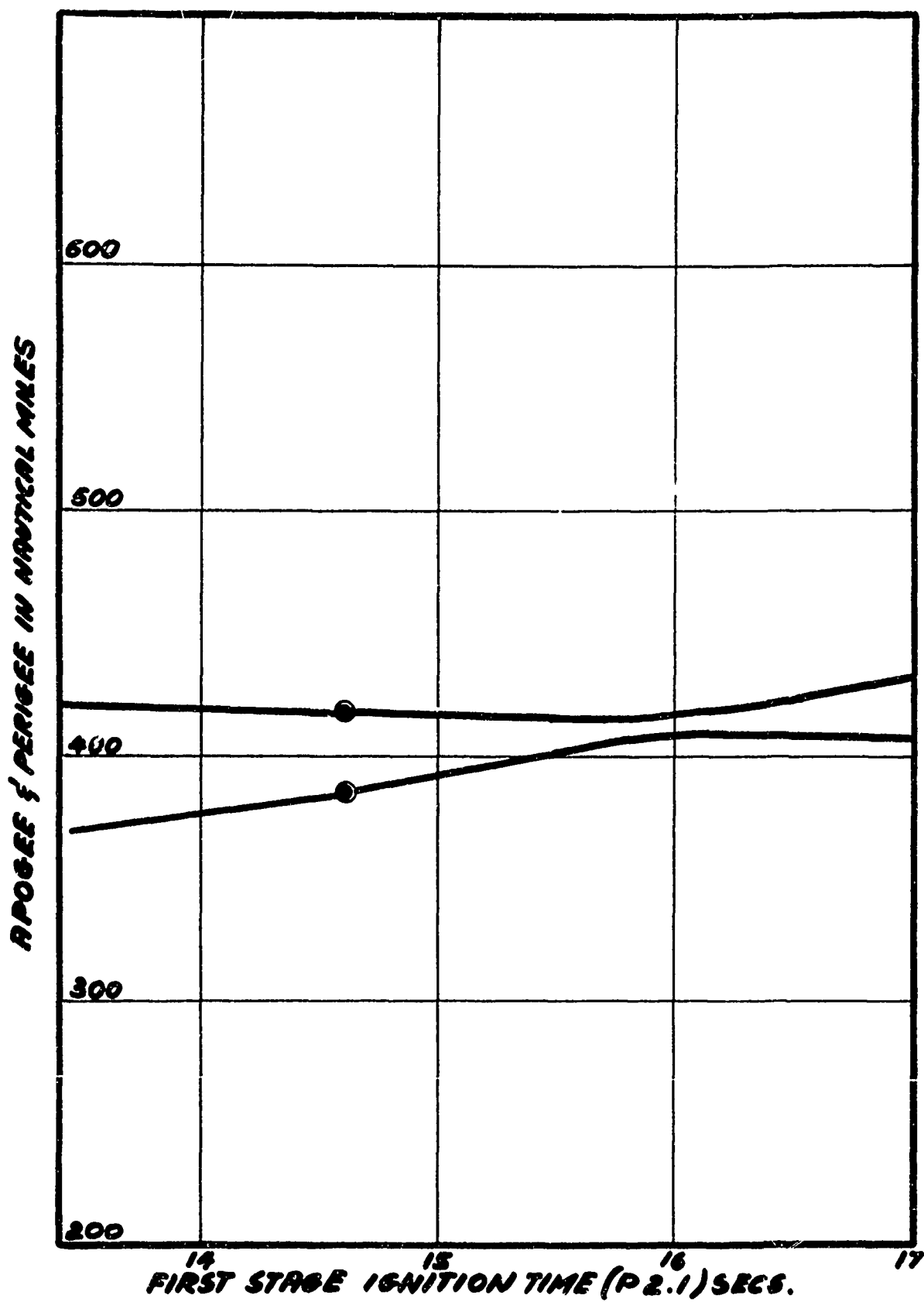


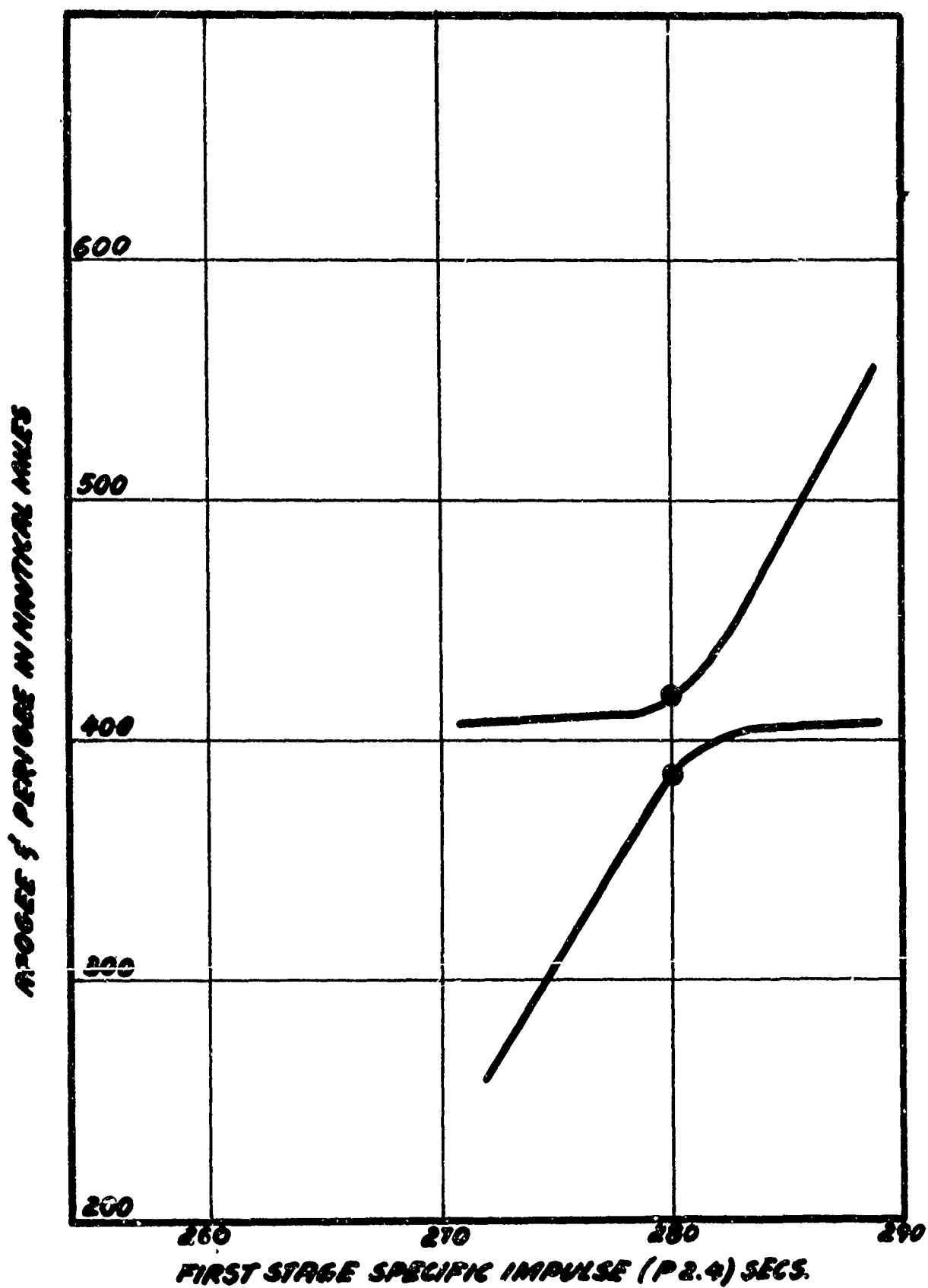
APPENDIX C

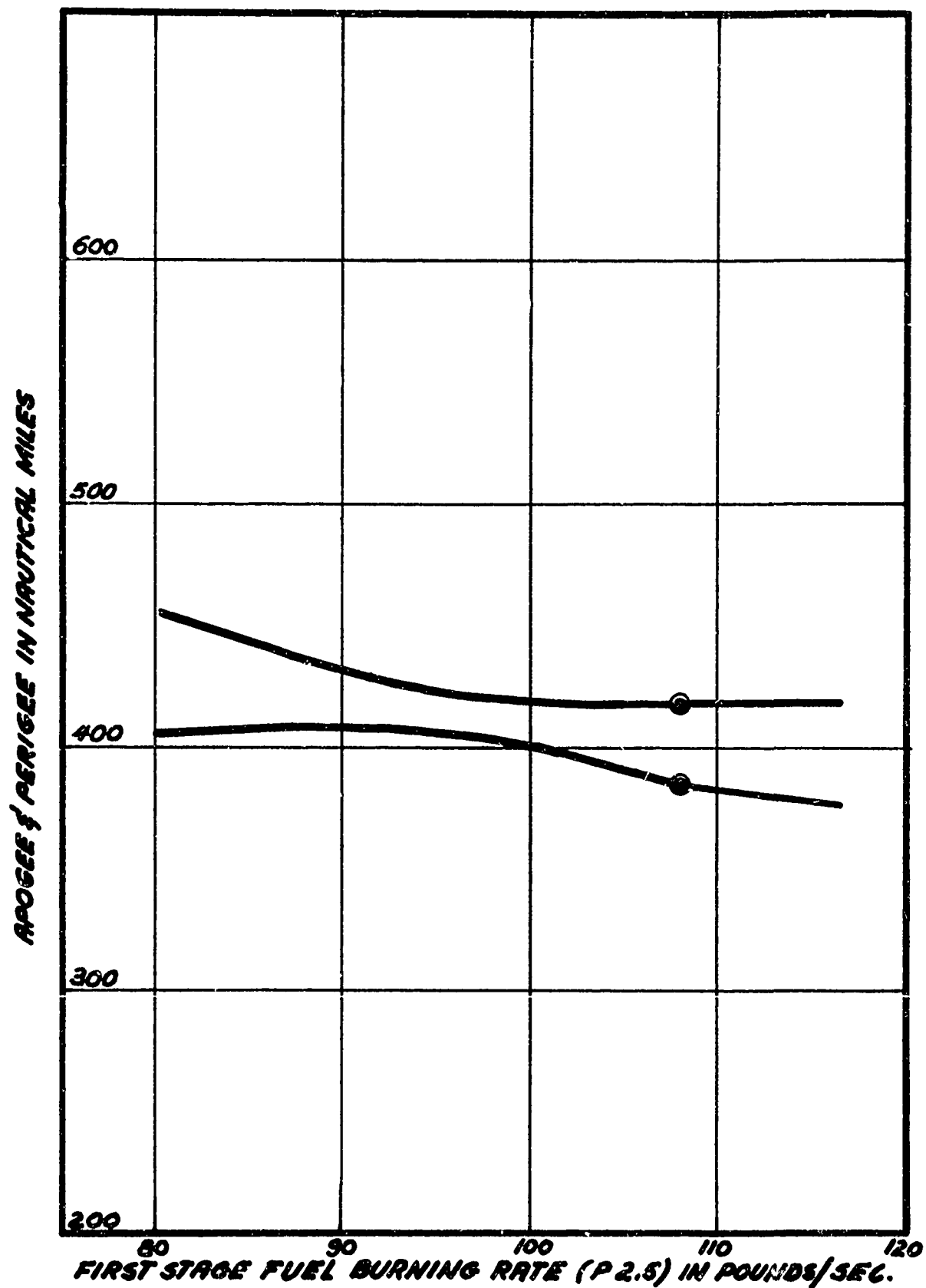
Graphical Summary of the
Sensitivity of the Orbit Parameter
to the Trajectory Parameter

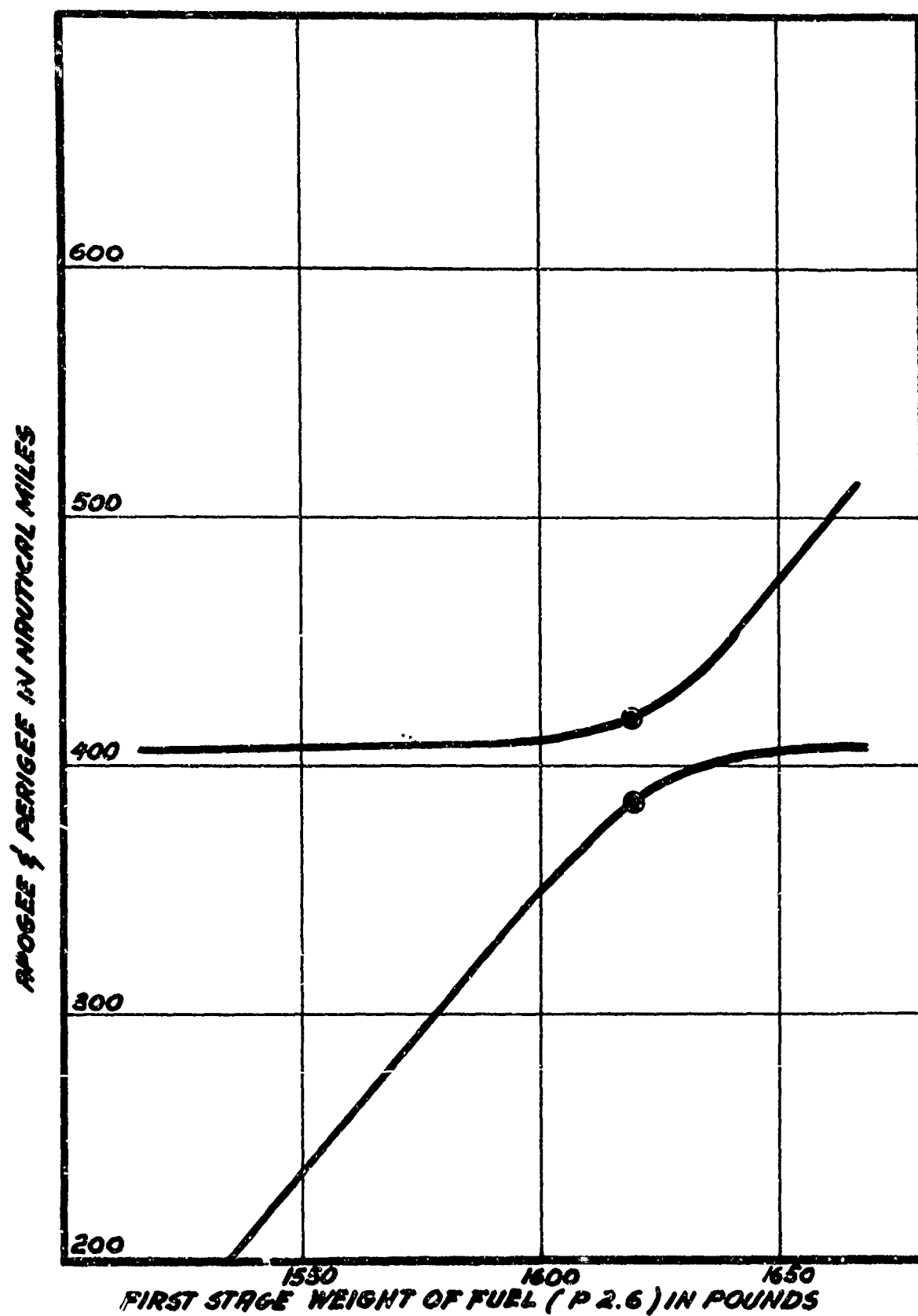




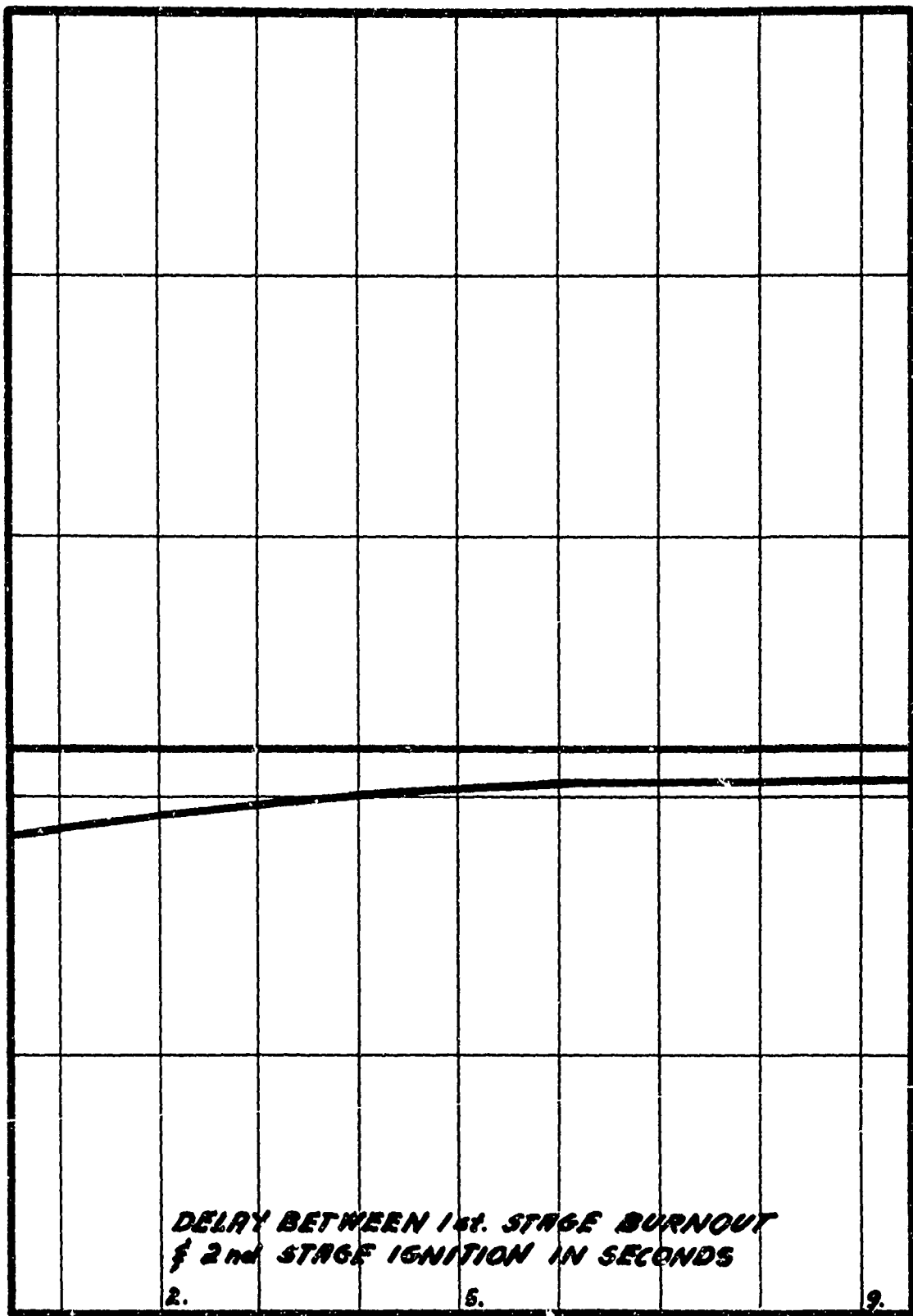




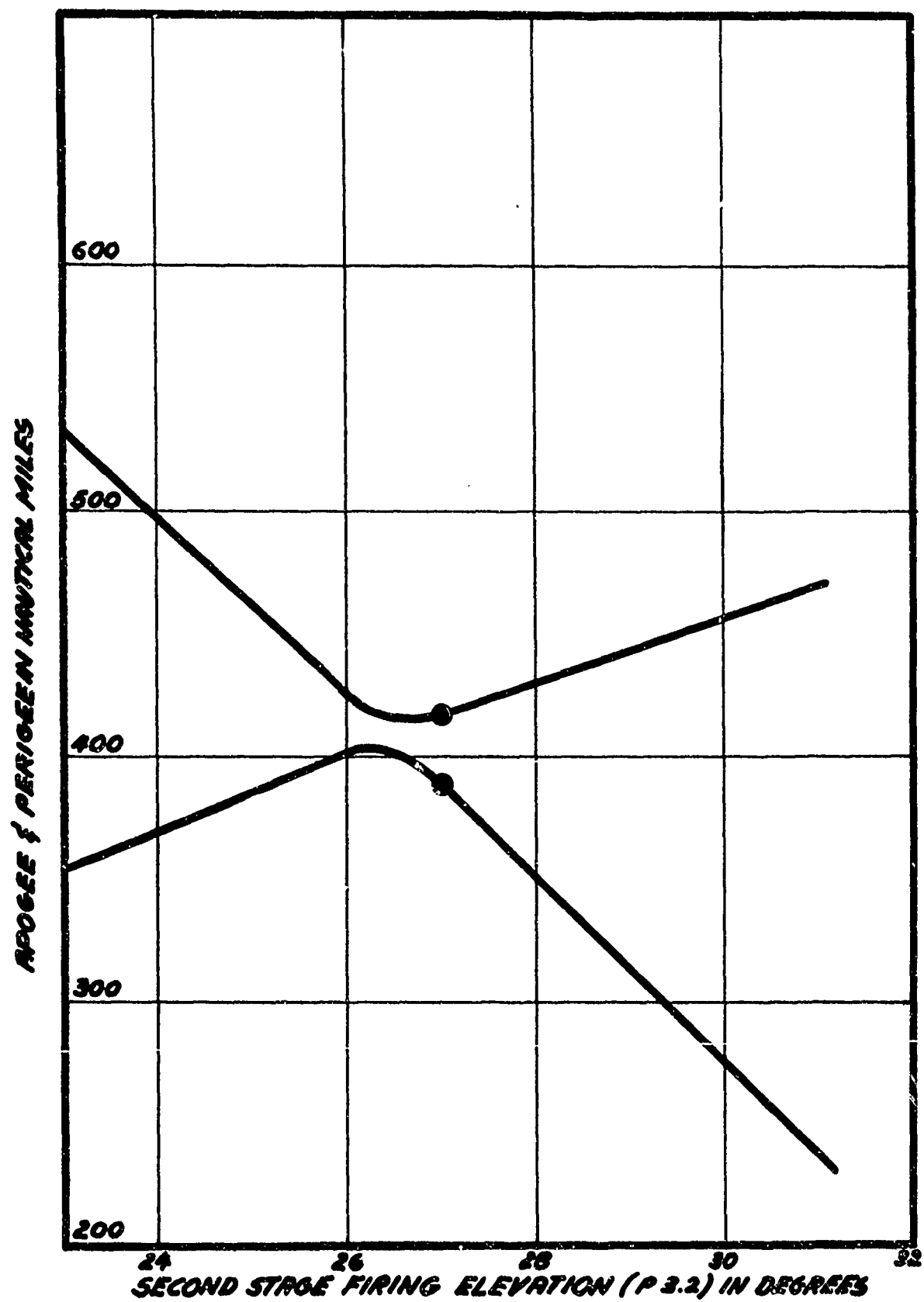


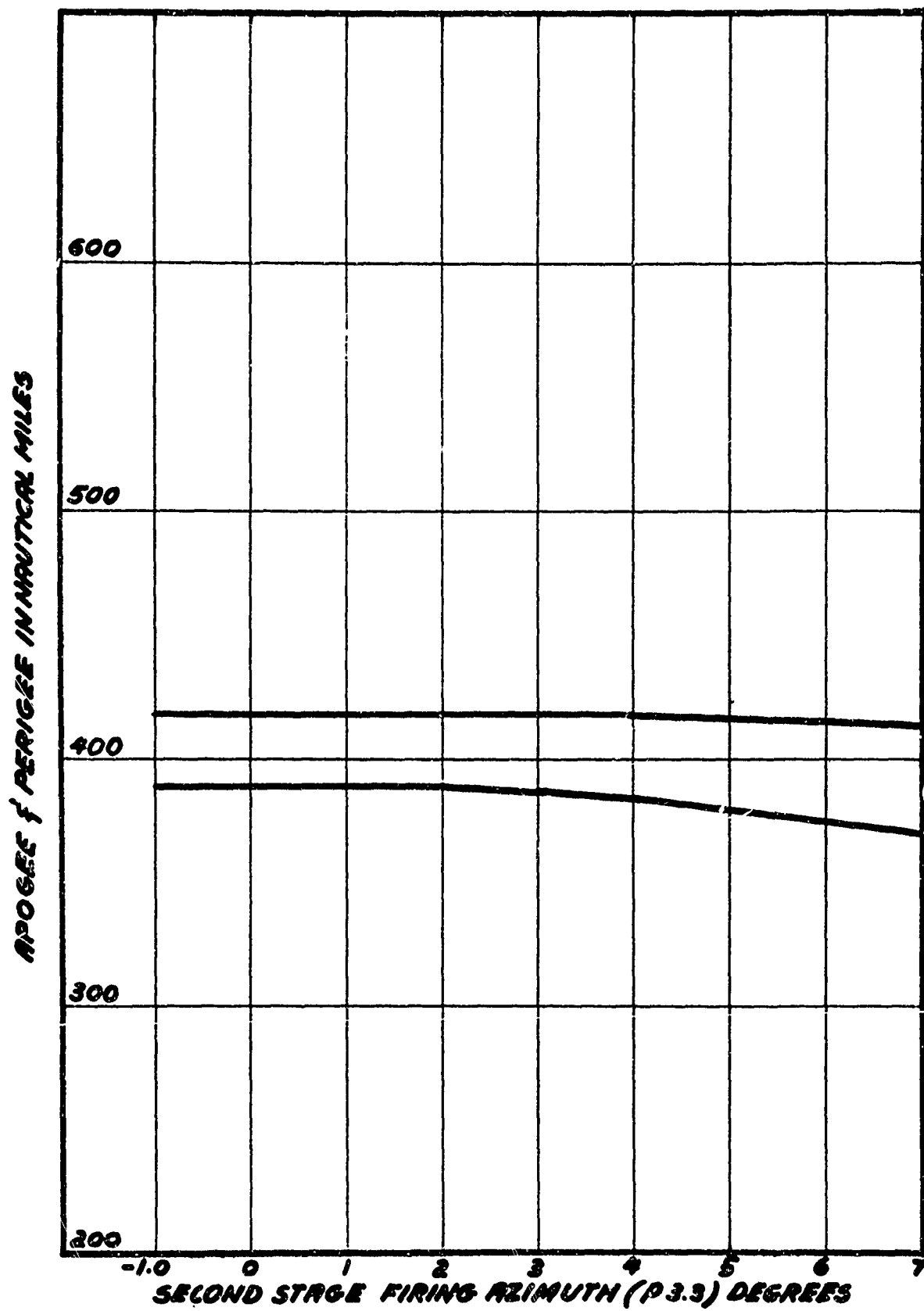


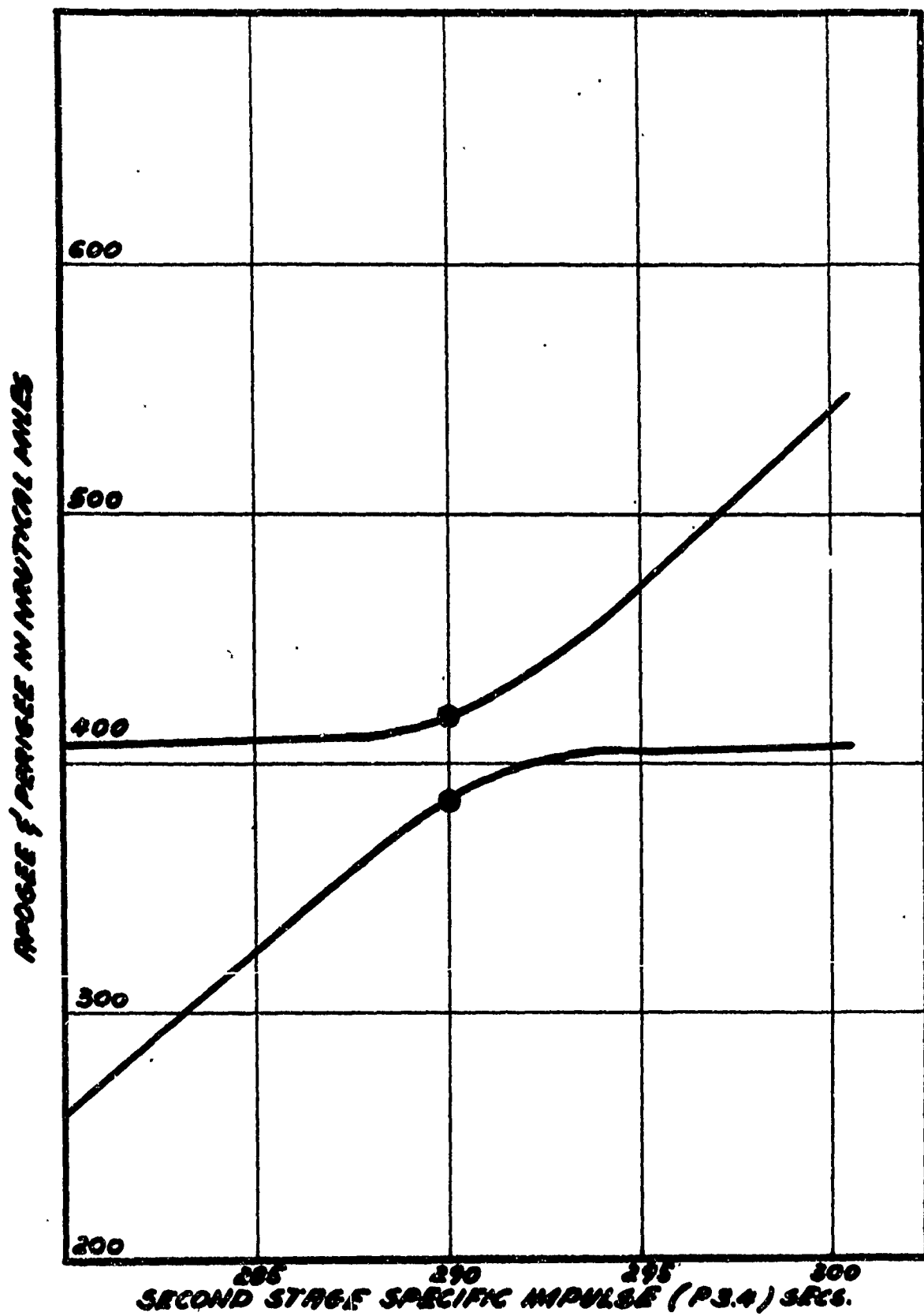
APOGEE & PERIGEE IN NAUTICAL MILES

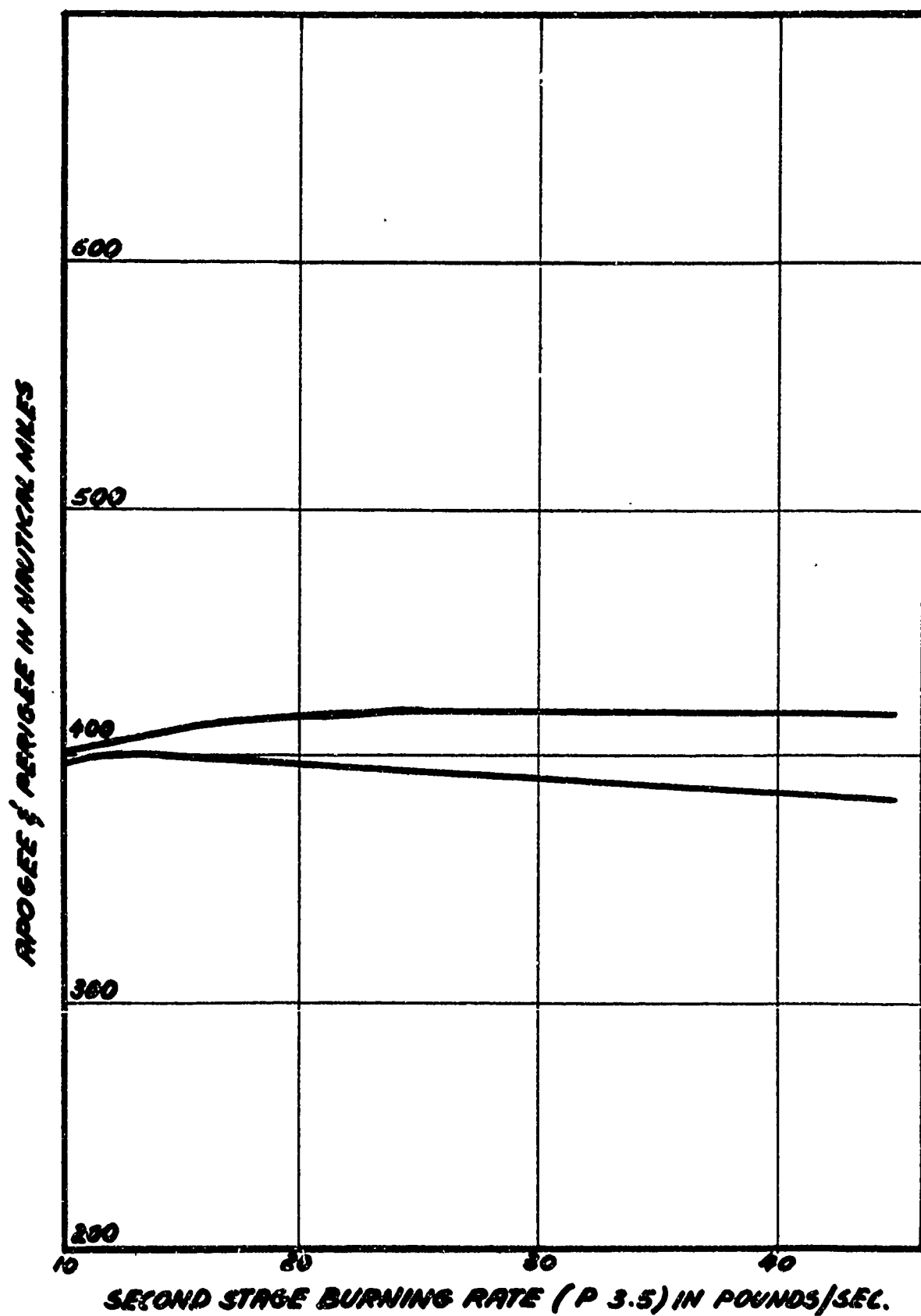


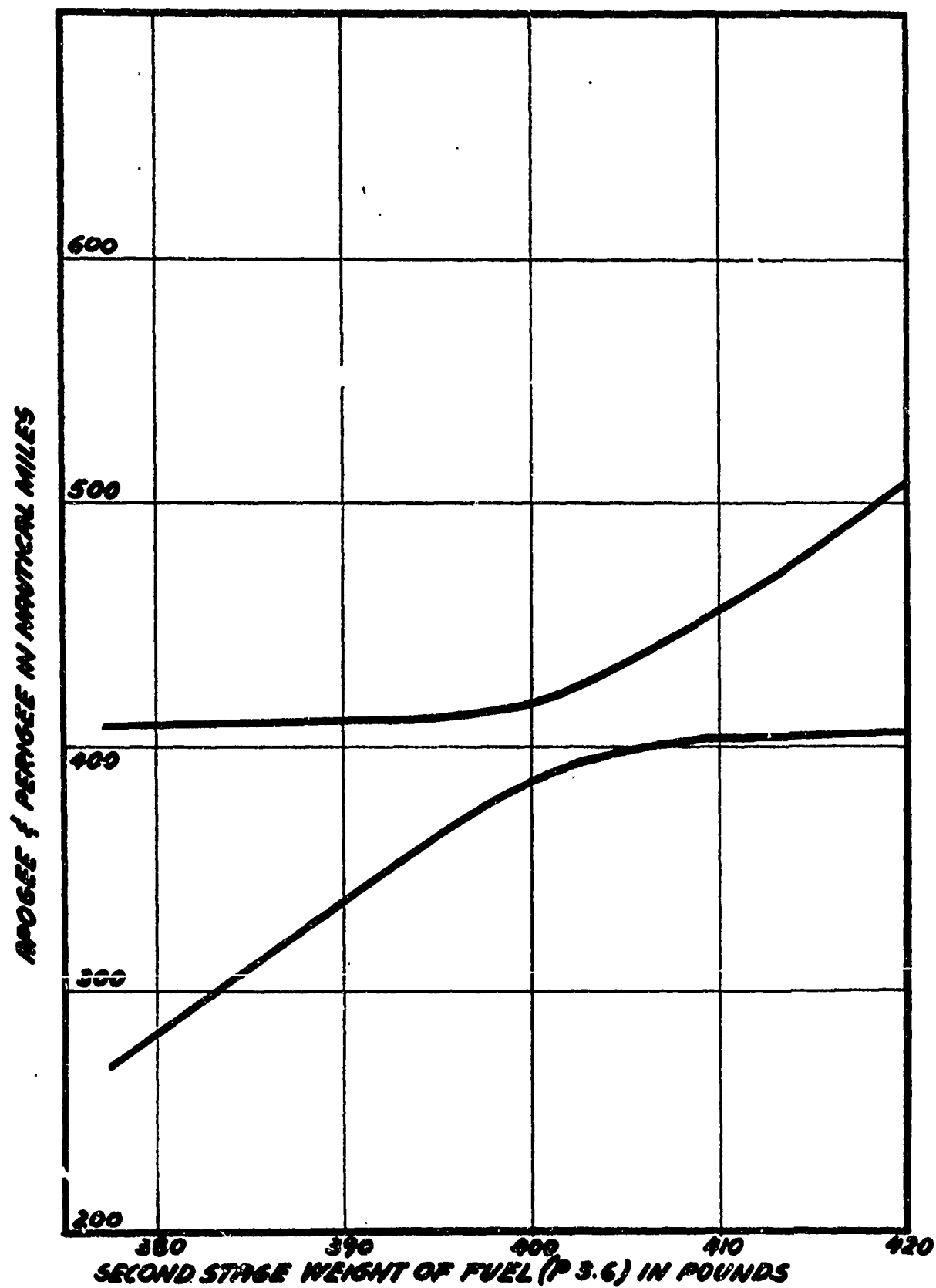
31.61 34.61 38.61
SECOND STAGE IGNITION TIME (P3.1) SECONDS

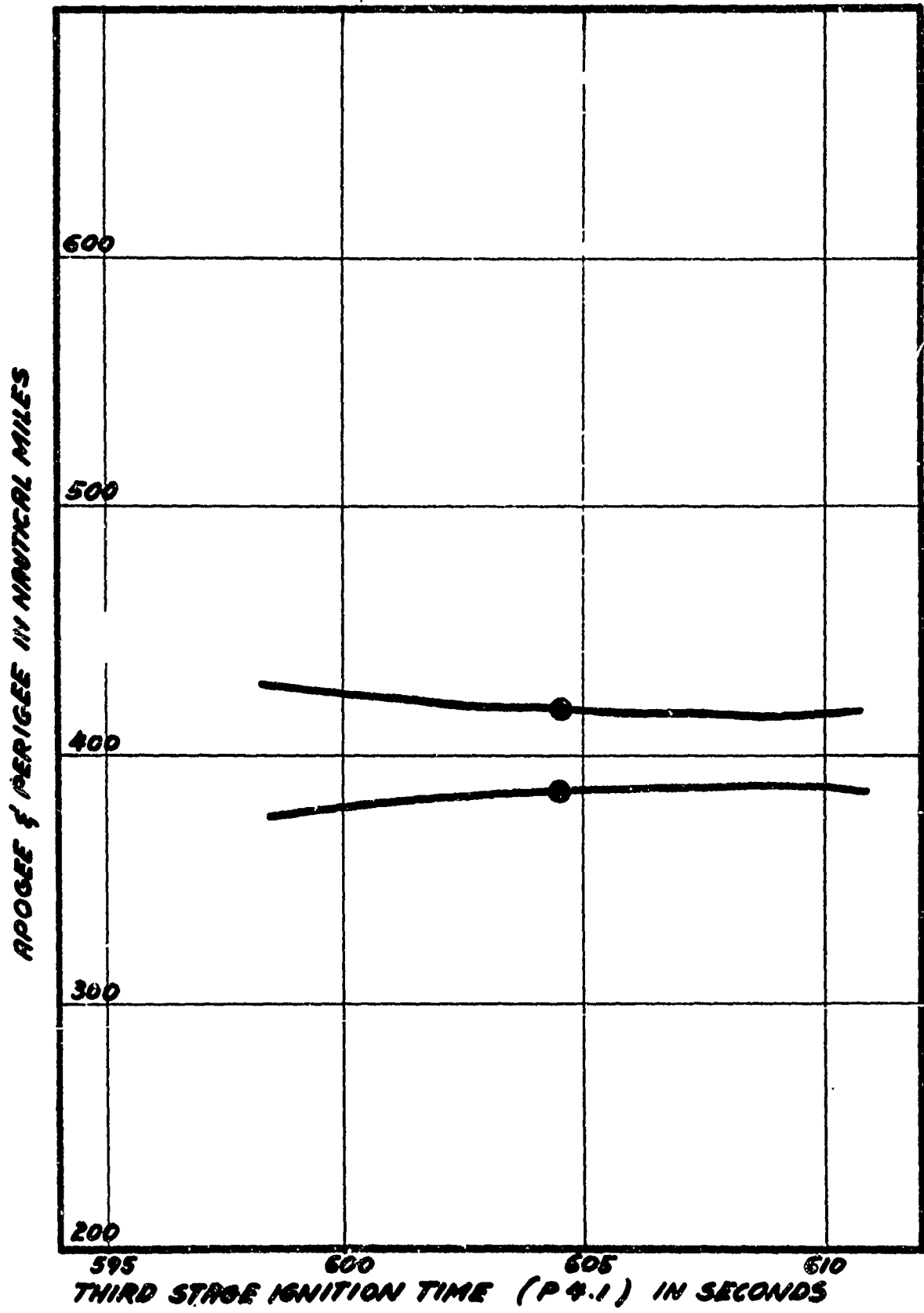


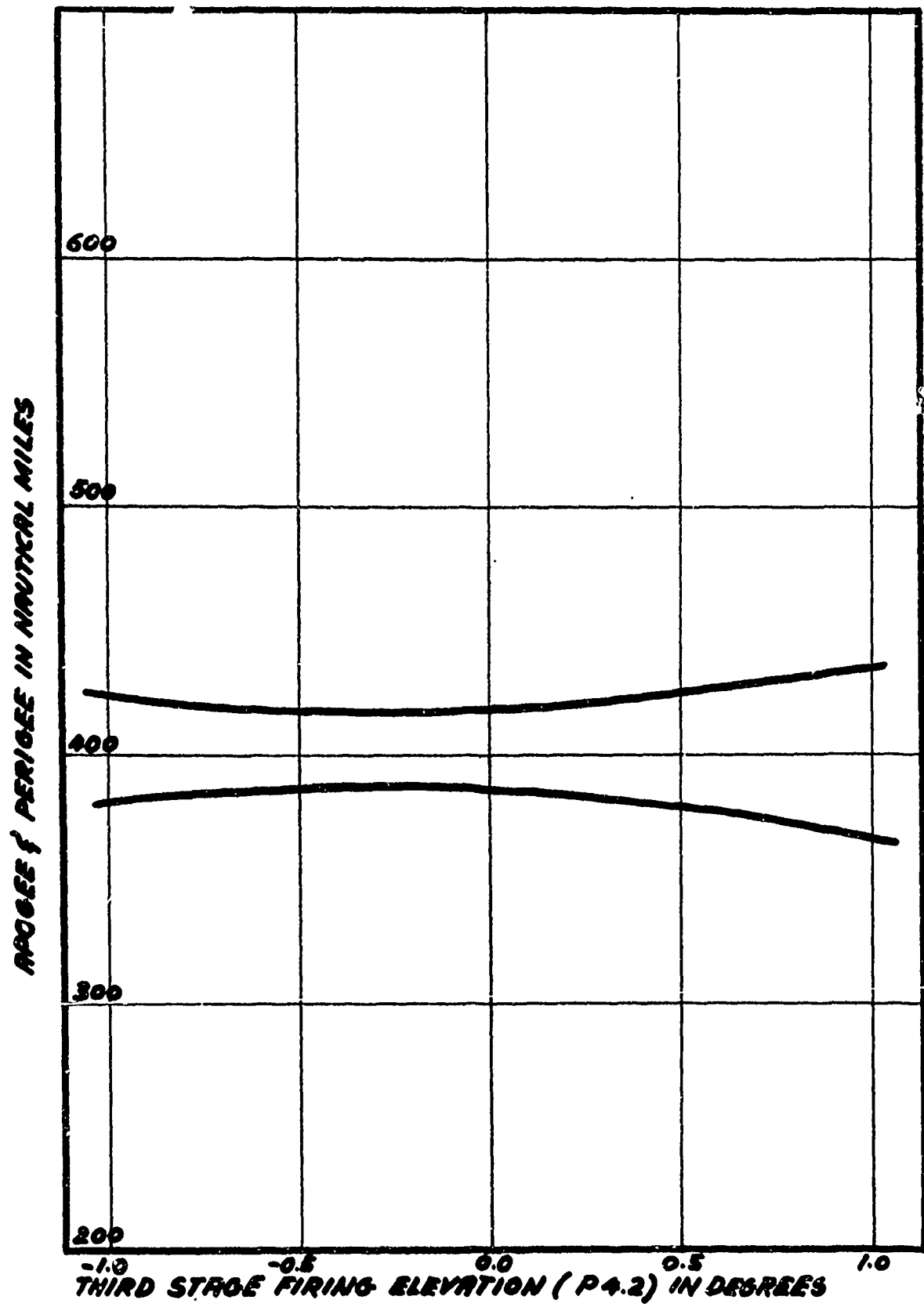


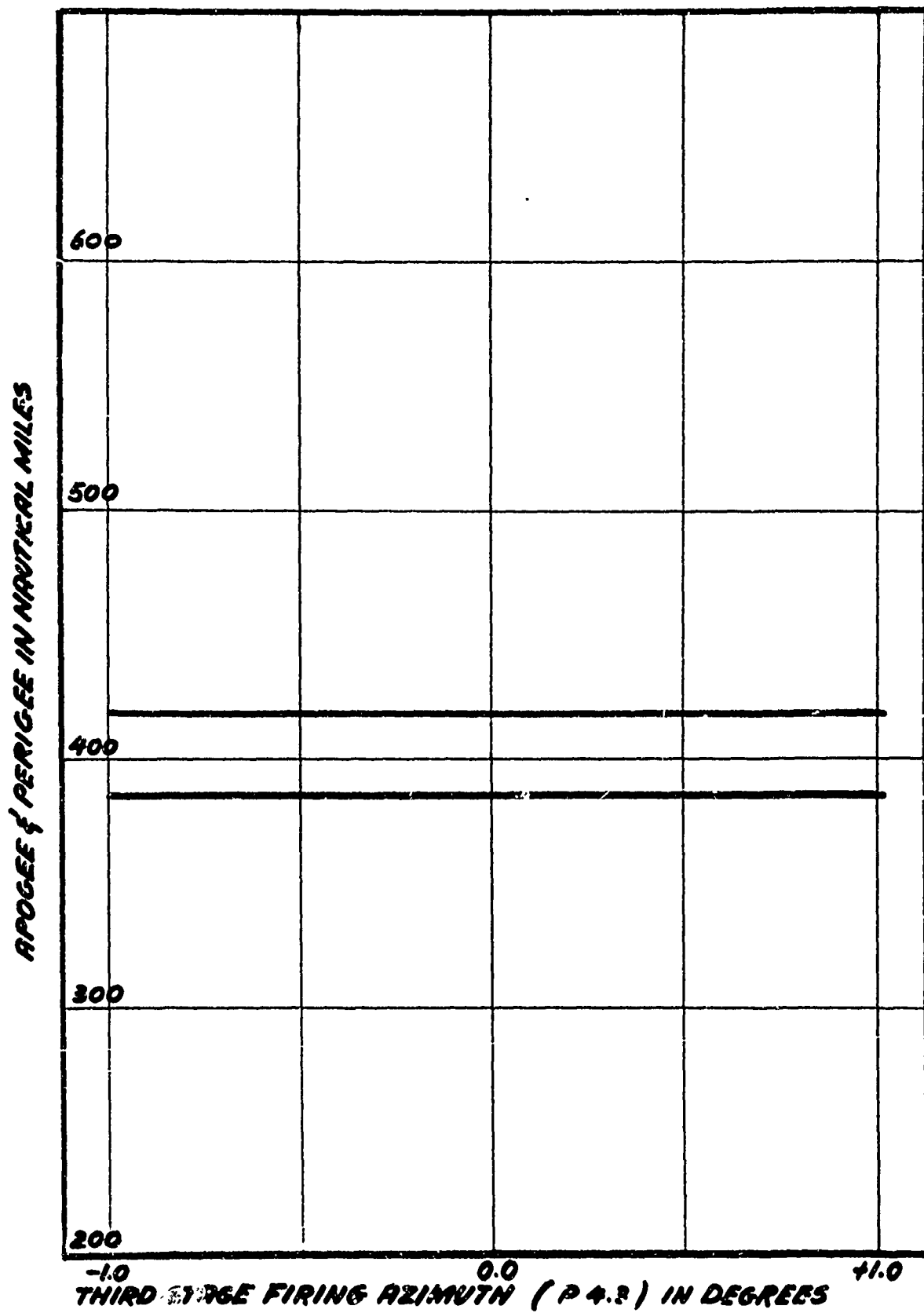


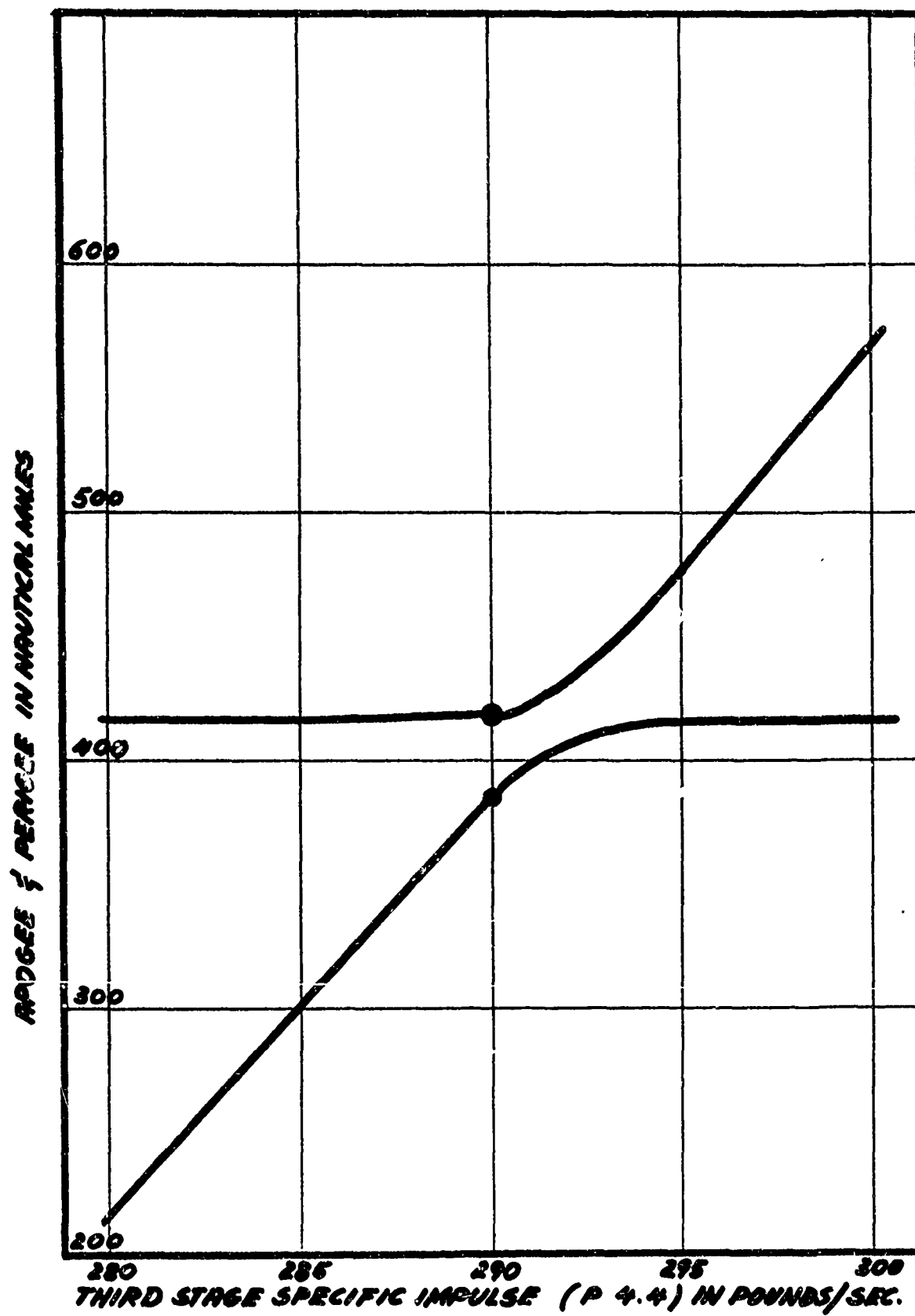


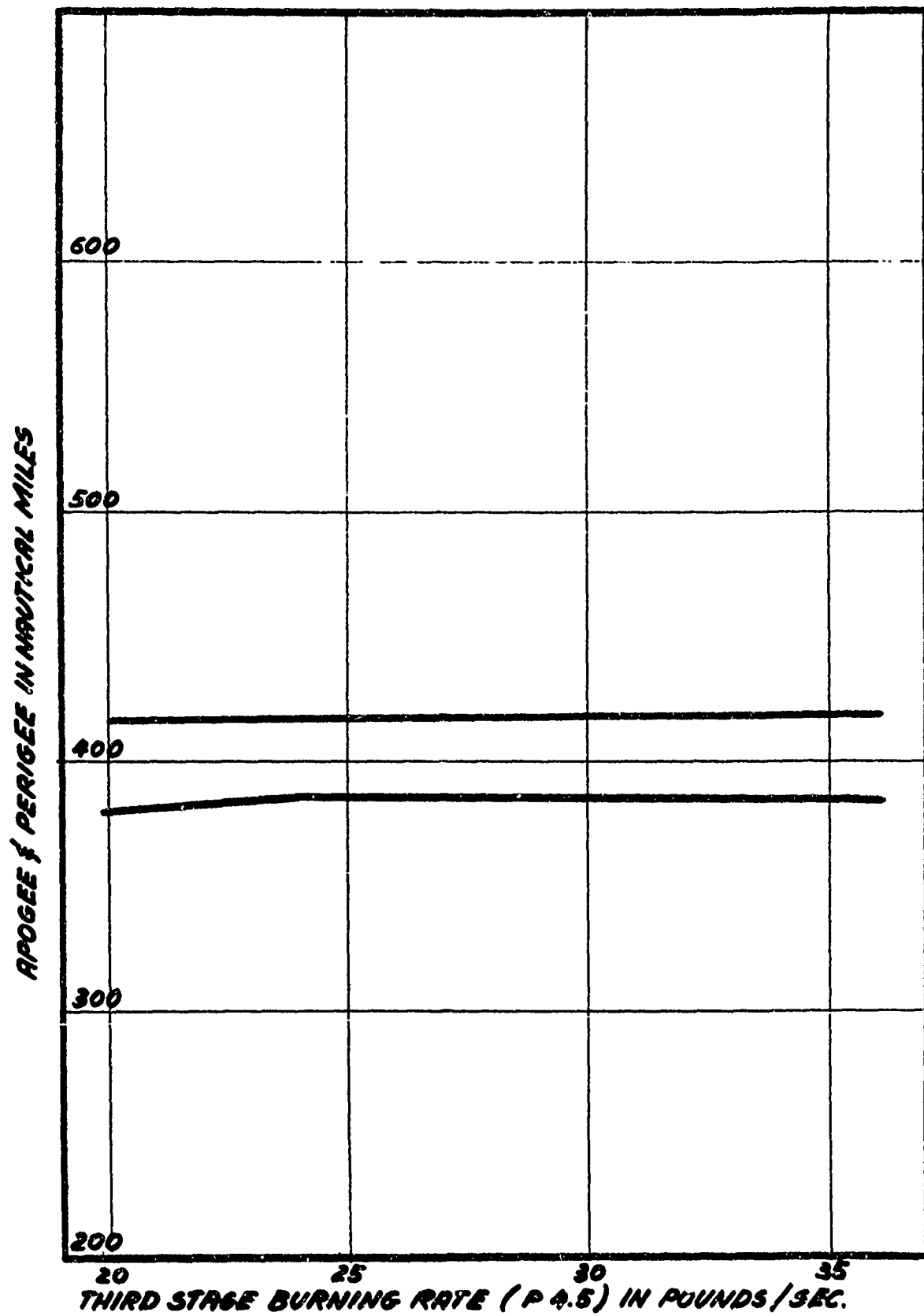


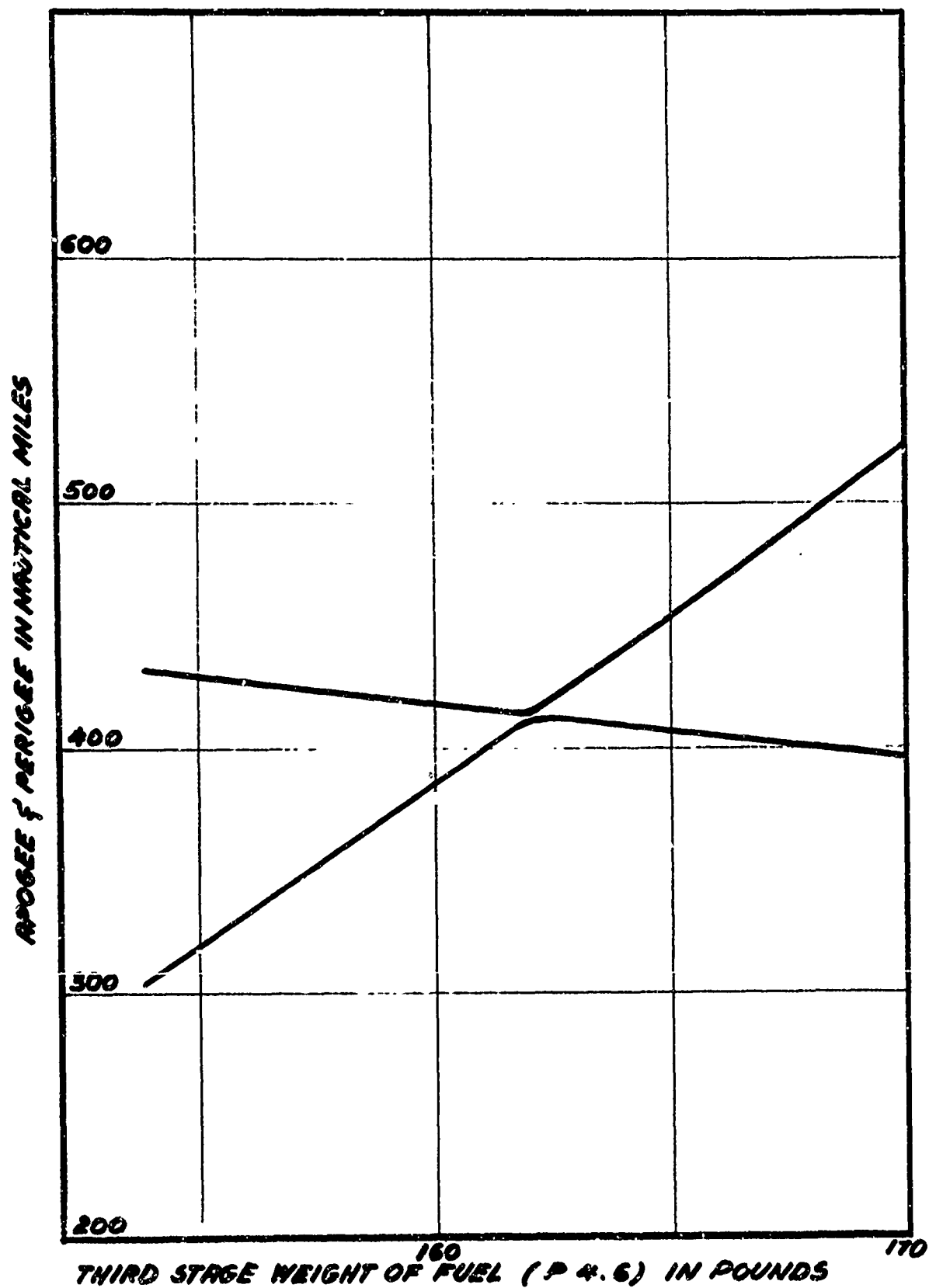


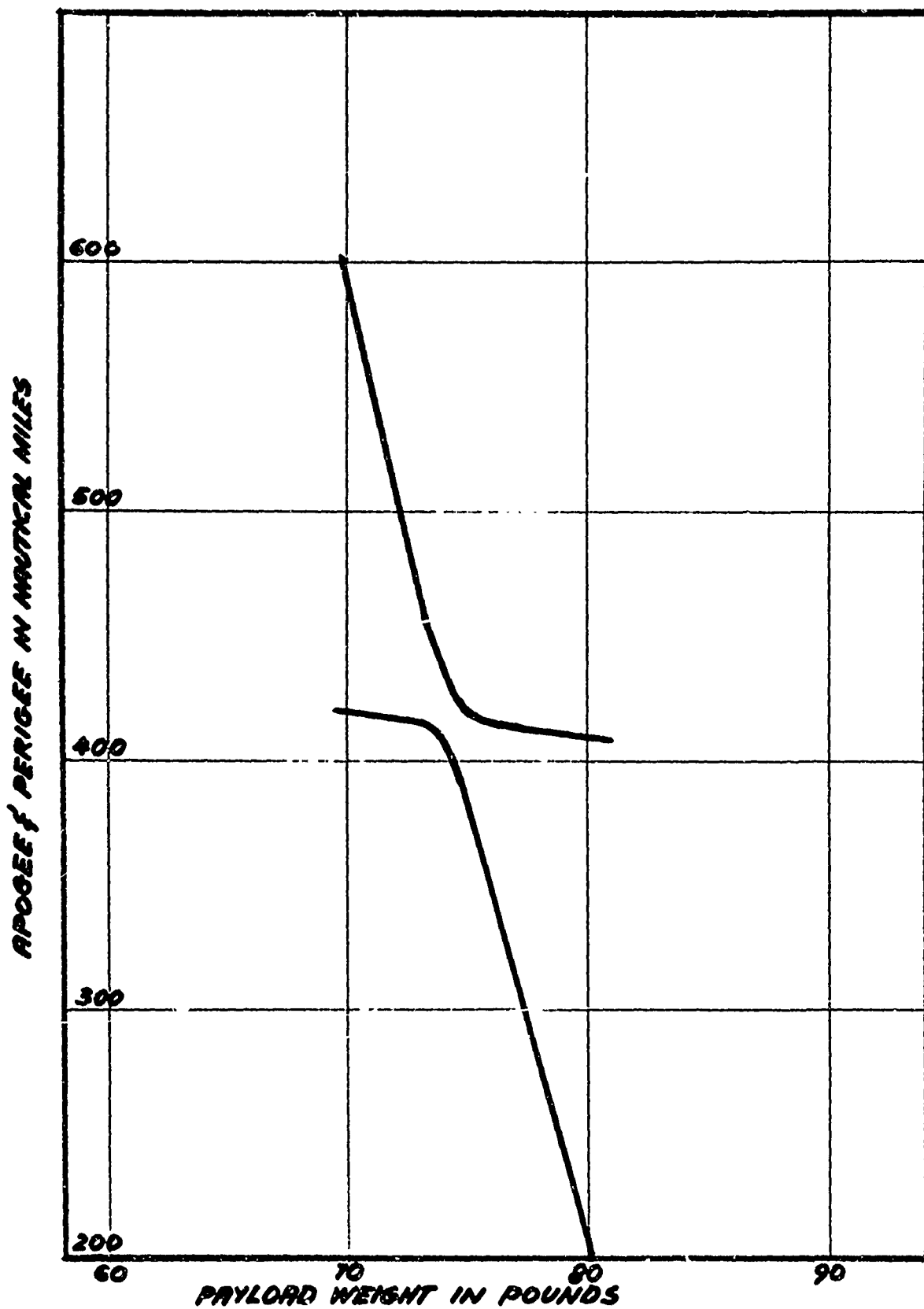












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13. ABSTRACT The orbit insertion trajectory for a Martlet 4 three stage gun-boosted rocket can be divided into four phases: gun launch and first glide, first stage burning, second stage burning, second glide, and third stage burning. Each phase is defined by a number of parameters which define a trajectory. The characteristics of a Martlet 4 trajectory are the large initial velocity and precise launch angle provided by the gun boost. A total of eighteen parameters can be identified with their associated probable errors. The influence of each individual error on a nominal trajectory leading to a 400 nautical mile orbit is studied. The orbit is found to be most sensitive to muzzle velocity, propellant specific impulse, fuel weight, elevation angle during motor burns, and payload weight. The assumed probable errors in each of these parameters are not likely to prevent a long time orbit.			

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